Real Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses signed numbers. By the end of the module students be able to distinguish between positive and negative real numbers, be able to read signed numbers and understand the origin and use of the double-negative product property.

Section Overview

- Positive and Negative Numbers
- Reading Signed Numbers
- Opposites
- The Double-Negative Property

Positive and Negative Numbers

Positive and Negative Numbers
Each real number other than zero has a **sign** associated with it. A real number is said to be a **positive number** if it is to the right of 0 on the number line and **negative** if it is to the left of 0 on the number line.

Note:
THE NOTATION OF SIGNED NUMBERS
+ and − Notation
A number is denoted as **positive** if it is directly preceded by a plus sign or no sign at all.
A number is denoted as **negative** if it is directly preceded by a minus sign.

Reading Signed Numbers

The plus and minus signs now have *two meanings*:

The plus sign can denote the operation of addition *or* a positive number.

The minus sign can denote the operation of subtraction *or* a negative number.

To avoid any confusion between "sign" and "operation," it is preferable to read the sign of a number as "positive" or "negative." When "+" is used as an operation sign, it is read as "plus." When "−" is used as an operation sign, it is read as "minus."

Sample Set A

Read each expression so as to avoid confusion between "operation" and "sign."

Example:
-8 should be read as "negative eight" rather than "minus eight."

Example:
$4 + (-2)$ should be read as "four plus negative two" rather than "four plus minus two."

Example:
$-6 + (-3)$should be read as "negative six plus negative three" rather than "minus six plus minus three."

Example:
$-15 - (-6)$should be read as "negative fifteen minus negative six" rather than "minus fifteen minus minus six."

Example:
$-5 + 7$ should be read as "negative five plus seven" rather than "minus five plus seven."

Example:
$0 - 2$ should be read as "zero minus two."

Practice Set A

Write each expression in words.

Exercise:

Problem: $6 + 1$

Solution:

six plus one

Exercise:

Problem: $2 + (-8)$

Solution:

two plus negative eight

Exercise:

Problem: $-7 + 5$

Solution:

negative seven plus five

Exercise:

Problem: $-10 - (+3)$

Solution:

negative ten minus three

Exercise:

Problem: $-1 - (-8)$

Solution:

negative one minus negative eight

Exercise:

Problem: $0 + (-11)$

Solution:

zero plus negative eleven

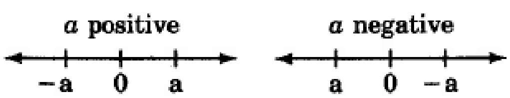
Opposites

Opposites
On the number line, each real number, other than zero, has an image on the opposite side of 0. For this reason, we say that each real number has an opposite. **Opposites** are the same distance from zero but have opposite signs.

The opposite of a real number is denoted by placing a negative sign directly in front of the number. Thus, if a is any real number, then $-a$ is its opposite.

Note: The letter "a" is a variable. Thus, "a" need not be positive, and "$-a$" need not be negative.

If a is any real number, $-a$ is opposite a on the number line.



The Double-Negative Property

The number a is opposite $-a$ on the number line. Therefore, $-(-a)$ is opposite $-a$ on the number line. This means that

$-(-a) = a$

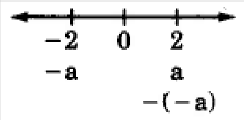
From this property of opposites, we can suggest the double-negative property for real numbers.

Double-Negative Property: $-(-a) = a$
If a is a real number, then
$-(-a) = a$

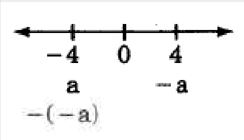
Sample Set B

Find the opposite of each number.

Example:
If $a = 2$, then $-a = -2$. Also, $-(-a) = -(-2) = 2$.



Example:
If $a = -4$, then $-a = -(-4) = 4$. Also, $-(-a) = a = -4$.



Practice Set B

Find the opposite of each number.
Exercise:

Problem: 8

Solution:

-8

Exercise:

Problem: 17

Solution:

-17

Exercise:

Problem: -6

Solution:

6

Exercise:

Problem: -15

Solution:

15

Exercise:

Problem: -(-1)

Solution:

-1

Exercise:

Problem: $-[-(-7)]$

Solution:

7

Exercise:

Problem: Suppose a is a positive number. Is $-a$ positive or negative?

Solution:

$-a$ is negative

Exercise:

Problem: Suppose a is a negative number. Is $-a$ positive or negative?

Solution:

$-a$ is positive

Exercise:

Problem:

Suppose we do not know the sign of the number k. Is $-k$ positive, negative, or do we not know?

Solution:

We must say that we do not know.

Exercises

Exercise:

Problem: A number is denoted as positive if it is directly preceded by .

Solution:

+ (or no sign)

Exercise:

Problem: A number is denoted as negative if it is directly preceded by .

How should the number in the following 6 problems be read? (Write in words.)
Exercise:

Problem: -7

Solution:

negative seven

Exercise:

Problem: -5

Exercise:

Problem: 15

Solution:

fifteen

Exercise:

Problem: 11

Exercise:

Problem: $-(-1)$

Solution:

negative negative one, or opposite negative one

Exercise:

Problem: $-(-5)$

For the following 6 problems, write each expression in words.

Exercise:

Problem: $5+3$

Solution:

five plus three

Exercise:

Problem: $3+8$

Exercise:

Problem: $15+(-3)$

Solution:

fifteen plus negative three

Exercise:

Problem: $1 + (-9)$

Exercise:

Problem: $-7 - (-2)$

Solution:

negative seven minus negative two

Exercise:

Problem: $0 - (-12)$

For the following 6 problems, rewrite each number in simpler form.

Exercise:

Problem: $-(-2)$

Solution:

2

Exercise:

Problem: $-(-16)$

Exercise:

Problem: $-[-(-8)]$

Solution:

-8

Exercise:

Problem: $-[-(-20)]$

Exercise:

Problem: $7-(-3)$

Solution:

$7+3=10$

Exercise:

Problem: $6-(-4)$

Exercises for Review

Exercise:

Problem: ([link]) Find the quotient; $8 \div 27$.

Solution:

0.296

Exercise:

Problem: ([link]) Solve the proportion: $\frac{5}{9}=\frac{60}{x}$

Exercise:

Problem: ([link]) Use the method of rounding to estimate the sum: $5829+8767$

Solution:

$6{,}000+9{,}000=15{,}000\ \ (5{,}829+8{,}767=14{,}596)$ or $5{,}800+8{,}800=14{,}600$

Exercise:

Problem: ([link]) Use a unit fraction to convert 4 yd to feet.

Exercise:

Problem: ([link]) Convert 25 cm to hm.

Solution:

0.0025 hm

Real Number Line
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The symbols, notations, and properties of numbers that form the basis of algebra, as well as exponents and the rules of exponents, are introduced in this chapter. Each property of real numbers and the rules of exponents are expressed both symbolically and literally. Literal explanations are included because symbolic explanations alone may be difficult for a student to interpret. Objectives of this module: be familiar with the real number line and the real numbers, understand the ordering of the real numbers.

Overview

- The Real Number Line
- The Real Numbers
- Ordering the Real Numbers

The Real Number Line

Real Number Line
In our study of algebra, we will use several collections of numbers. The **real number line** allows us to **visually** display the numbers in which we are interested.

A line is composed of infinitely many points. To each point we can associate a unique number, and with each number we can associate a particular point.

Coordinate
The number associated with a point on the number line is called the **coordinate** of the point.

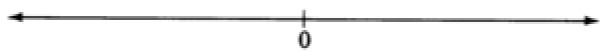
Graph
The point on a line that is associated with a particular number is called the **graph** of that number.

We construct the real number line as follows:

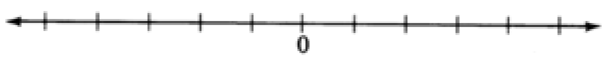
Construction of the Real Number Line

1. Draw a horizontal line.

2. Choose any point on the line and label it 0. This point is called the **origin**.



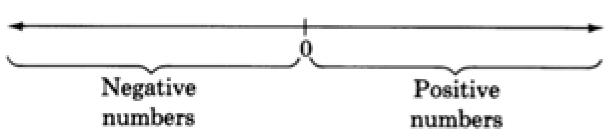
3. Choose a convenient length. This length is called "1 unit." Starting at 0, mark this length off in both directions, being careful to have the lengths look like they are about the same.



We now define a real number.

Real Number
A **real number** is any number that is the coordinate of a point on the real number line.

Positive and Negative Real Numbers
The collection of these infinitely many numbers is called the **collection of real numbers**. The real numbers whose graphs are to the right of 0 are called the **positive real numbers**. The real numbers whose graphs appear to the left of 0 are called the **negative real numbers**.

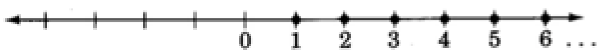


The number 0 is neither positive nor negative.

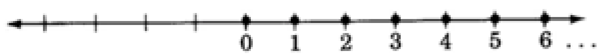
The Real Numbers

The collection of real numbers has many subcollections. The subcollections that are of most interest to us are listed below along with their notations and graphs.

Natural Numbers
The **natural numbers** (N)**:** $\{1,\ 2, 3,\ \ldots\}$

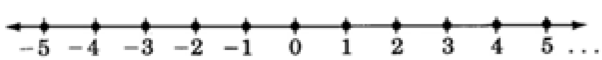


Whole Numbers
The **whole numbers** (W)**:** $\{0,\ 1,\ 2, 3,\ \ldots\}$



Notice that every natural number is a whole number.

Integers
The **integers** (Z)**:** $\{\ldots,\ -3, -2, -1, 0, 1,\ 2,\ 3,\ \ldots\}$



Notice that every whole number is an integer.

Rational Numbers
The **rational numbers** (Q)**:** Rational numbers are real numbers that can be written in the form a/b, where a and b are integers, and $b \neq 0$.

Fractions
Rational numbers are commonly called **fractions.**

Division by 1

Since b can equal 1, every integer is a rational number: $\frac{a}{1} = a$.

Division by 0
Recall that $10/2 = 5$ since $2 \cdot 5 = 10$. However, if $10/0 = x$, then $0 \cdot x = 10$. But $0 \cdot x = 0$, not 10. This suggests that no quotient exists.

Now consider $0/0 = x$. If $0/0 = x$, then $0 \cdot x = 0$. But this means that x could be any number, that is, $0/0 = 4$ since $0 \cdot 4 = 0$, or $0/0 = 28$ since $0 \cdot 28 = 0$. This suggests that the quotient is indeterminant.

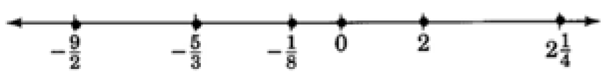
$x/0$ Is Undefined or Indeterminant
Division by 0 is undefined or indeterminant.

Do not divide by 0.

Rational numbers have decimal representations that either terminate or do not terminate but contain a repeating block of digits. Some examples are:

$$\underset{\text{Terminating}}{\frac{3}{4} = 0.75} \qquad \underset{\text{Nonterminating, but repeating}}{\frac{15}{11} = 1.36363636\ldots}$$

Some rational numbers are graphed below.



Irrational Numbers
The **irrational numbers** (Ir)**:** Irrational numbers are numbers that cannot be written as the quotient of two integers. They are numbers whose decimal representations are nonterminating and nonrepeating. Some examples are

$$4.01001000100001\ldots \qquad \pi = 3.1415927\ldots$$

Notice that the collections of rational numbers and irrational numbers have no numbers in common.

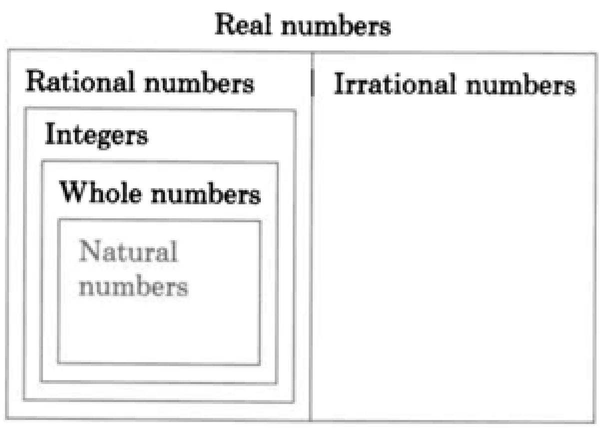
When graphed on the number line, the rational and irrational numbers account for every point on the number line. Thus each point on the number

line has a coordinate that is either a rational or an irrational number.

In summary, we have

Sample Set A

The summaray chart illustrates that



Example:
Every natural number is a real number.

Example:
Every whole number is a real number.

Example:
No integer is an irrational number.

Practice Set A

Exercise:

Problem: Is every natural number a whole number?

Solution:

yes

Exercise:

Problem: Is every whole number an integer?

Solution:

yes

Exercise:

Problem: Is every integer a rational number?

Solution:

yes

Exercise:

Problem: Is every rational number a real number?

Solution:

yes

Exercise:

Problem: Is every integer a natural number?

Solution:

no

Exercise:

Problem: Is there an integer that is a natural number?

Solution:

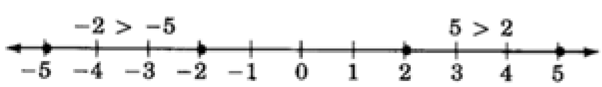
yes

Ordering the Real Numbers

Ordering the Real Numbers
A real number b is said to be greater than a real number a, denoted $b > a$, if the graph of b is to the right of the graph of a on the number line.

Sample Set B

As we would expect, $5 > 2$ since 5 is to the right of 2 on the number line. Also, $-2 > -5$ since -2 is to the right of -5 on the number line.



Practice Set B

Exercise:

Problem: Are all positive numbers greater than 0?

Solution:

yes

Exercise:

Problem: Are all positive numbers greater than all negative numbers?

Solution:

yes

Exercise:

Problem: Is 0 greater than all negative numbers?

Solution:

yes

Exercise:

Problem:

Is there a largest positive number? Is there a smallest negative number?

Solution:

no, no

Exercise:

Problem:

How many real numbers are there? How many real numbers are there between 0 and 1?

Solution:

infinitely many, infinitely many

Sample Set C

Example:
What integers can replace x so that the following statement is true?

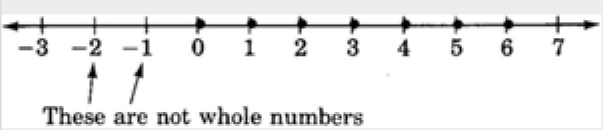
$$-4 \leq x < 2$$

This statement indicates that the number represented by x is between -4 and 2. Specifically, -4 is less than or equal to x, and at the same time, x is strictly less than 2. This statement is an example of a compound inequality.

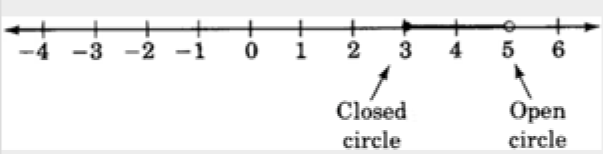
The integers are $-4, \ -3, \ -2, \ -1, \ 0, \ 1.$

Example:
Draw a number line that extends from -3 to 7. Place points at all whole numbers between and including -2 and 6.

Example:
Draw a number line that extends from -4 to 6 and place points at all real numbers greater than or equal to 3 but strictly less than 5.



It is customary to use a **closed circle** to indicate that a point is included in the graph and an **open circle** to indicate that a point is not included.



Practice Set C

Exercise:

Problem:

What whole numbers can replace x so that the following statement is true?

$-3 \le x < 3$

Solution:

0, 1, 2

Exercise:

Problem:

Draw a number line that extends from -5 to 3 and place points at all numbers greater than or equal to -4 but strictly less than 2.

Solution:

Exercises

For the following problems, next to each real number, note all collections to which it belongs by writing N for natural numbers, W for whole numbers, Z for integers, Q for rational numbers, Ir for irrational numbers, and R for real numbers. Some numbers may require more than one letter.

Exercise:

Problem: $\frac{1}{2}$

Solution:

$Q,\ R$

Exercise:

Problem: -12

Exercise:

Problem: 0

Solution:

$W,\ Z,\ Q,\ R$

Exercise:

Problem: $-24\frac{7}{8}$

Exercise:

Problem: $86.3333\ldots$

Solution:

$Q,\ R$

Exercise:

Problem: $49.125125125\ldots$

Exercise:

Problem: -15.07

Solution:

Q, R

For the following problems, draw a number line that extends from -3 to 3. Locate each real number on the number line by placing a point (closed circle) at its approximate location.

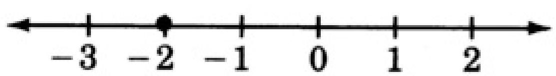
Exercise:

Problem: $1\frac{1}{2}$

Exercise:

Problem: -2

Solution:



Exercise:

Problem: $-\frac{1}{8}$

Exercise:

Problem: Is 0 a positive number, negative number, neither, or both?

Solution:

neither

Exercise:

Problem:

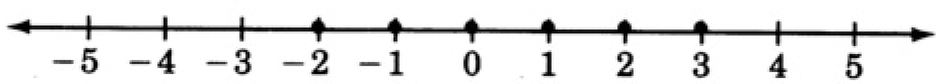
An integer is an even integer if it can be divided by 2 without a remainder; otherwise the number is odd. Draw a number line that extends from -5 to 5 and place points at all negative even integers and at all positive odd integers.

Exercise:

Problem:

Draw a number line that extends from -5 to 5. Place points at all integers strictly greater than -3 but strictly less than 4.

Solution:



For the following problems, draw a number line that extends from -5 to 5. Place points at all real numbers between and including each pair of numbers.

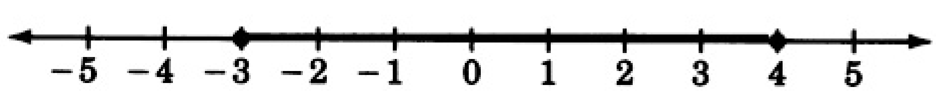
Exercise:

Problem: -5 and -2

Exercise:

Problem: -3 and 4

Solution:



Exercise:

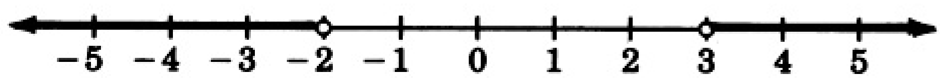
Problem: -4 and 0

Exercise:

Problem:

Draw a number line that extends from -5 to 5. Is it possible to locate any numbers that are strictly greater than 3 but also strictly less than -2?

Solution:



; no

For the pairs of real numbers shown in the following problems, write the appropriate relation symbol $(<, \ >, \ =)$ in place of the $*$.

Exercise:

Problem: $-5 * -1$

Exercise:

Problem: $-3 * 0$

Solution:

$<$

Exercise:

Problem: $-4 * 7$

Exercise:

Problem: $6 * -1$

Solution:

$>$

Exercise:

Problem: $-\frac{1}{4} * -\frac{3}{4}$

Exercise:

Problem: Is there a largest real number? If so, what is it?

Solution:

no

Exercise:

Problem: Is there a largest integer? If so, what is it?

Exercise:

Problem: Is there a largest two-digit integer? If so, what is it?

Solution:

99

Exercise:

Problem: Is there a smallest integer? If so, what is it?

Exercise:

Problem: Is there a smallest whole number? If so, what is it?

Solution:

yes, 0

For the following problems, what numbers can replace x so that the following statements are true?

Exercise:

Problem: $-1 \leq x \leq 5 \quad x \text{ an integer}$

Exercise:

Problem: $-7 < x < -1, \quad x \text{ an integer}$

Solution:

$-6, \ -5, \ -4, \ -3, \ -2$

Exercise:

Problem: $-3 \leq x \leq 2, \quad x \text{ a natural number}$

Exercise:

Problem: $-15 < x \leq -1, \quad x \text{ a natural number}$

Solution:

There are no natural numbers between −15 and −1.

Exercise:

Problem: $-5 \leq x < 5, \quad x \text{ a whole number}$

Exercise:

Problem:

The temperature in the desert today was ninety-five degrees. Represent this temperature by a rational number.

Solution:

$\left(\frac{95}{1}\right)^{\circ}$

Exercise:

Problem:

The temperature today in Colorado Springs was eight degrees below zero. Represent this temperature with a real number.

Exercise:

Problem: Is every integer a rational number?

Solution:

Yes, every integer is a rational number.

Exercise:

Problem: Is every rational number an integer?

Exercise:

Problem:

Can two rational numbers be added together to yield an integer? If so, give an example.

Solution:

Yes. $\frac{1}{2} + \frac{1}{2} = 1 or 1 + 1 = 2$

For the following problems, on the number line, how many units (intervals) are there between?

Exercise:

Problem: 0 and 2?

Exercise:

Problem: -5 and 0?

Solution:

5 units

Exercise:

Problem: 0 and 6?

Exercise:

Problem: -8 and 0?

Solution:

8 units

Exercise:

Problem: -3 and 4?

Exercise:

Problem: m and n, $m > n$?

Solution:

$m - n$ units

Exercise:

Problem: $-a$ and $-b$, $-b > -a$?

Exercises for Review

Exercise:

Problem: **([link])** Find the value of $6 + 3(15 - 8) - 4$.

Solution:

23

Exercise:

Problem: **([link])** Find the value of $5(8 - 6) + 3(5 + 2 \cdot 3)$.

Exercise:

Problem:

([link]) Are the statements $y < 4$ and $y \geq 4$ the same or different?

Solution:

different

Exercise:

Problem:

([link]) Use algebraic notation to write the statement "six times a number is less than or equal to eleven."

Exercise:

Problem:

([link]) Is the statement $8(15 - 3 \cdot 4) - 3 \cdot 7 \geq 3$ true or false?

Solution:

true

Absolute Value
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses absolute value. By the end of the module students should understand the geometric and algebraic definitions of absolute value.

Section Overview

- Geometric Definition of Absolute Value
- Algebraic Definition of Absolute Value

Geometric Definition of Absolute Value

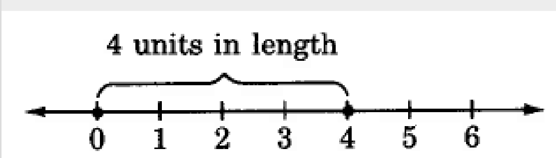
Absolute Value-Geometric Approach
Geometric definition of absolute value:
The **absolute value** of a number a, denoted $\mid a \mid$, is the distance from a to 0 on the number line.

Absolute value answers the question of "how far," and not "which way." The phrase "how far" implies "length" and *length is always a nonnegative quantity*. Thus, the absolute value of a number is a nonnegative number.

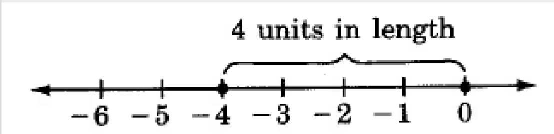
Sample Set A

Determine each value.

Example:
$\mid 4 \mid = 4$



Example:
$|-4| = 4$



Example:
$|0| = 0$

Example:
$-|5| = -5$. The quantity on the left side of the equal sign is read as "negative the absolute value of 5." The absolute value of 5 is 5. Hence, negative the absolute value of 5 is -5.

Example:
$-|-3| = -3$. The quantity on the left side of the equal sign is read as "negative the absolute value of -3." The absolute value of -3 is 3. Hence, negative the absolute value of -3 is $-(3) = -3$.

Practice Set A

By reasoning geometrically, determine each absolute value.

Exercise:

Problem: $|7|$

Solution:

7

Exercise:

Problem: $\mid -3 \mid$

Solution:

3

Exercise:

Problem: $\mid 12 \mid$

Solution:

12

Exercise:

Problem: $\mid 0 \mid$

Solution:

0

Exercise:

Problem: $- \mid 9 \mid$

Solution:

-9

Exercise:

Problem: $- \mid -6 \mid$

Solution:

-6

Algebraic Definition of Absolute Value

From the problems in [link], we can suggest the following algebraic definition of absolute value. Note that the definition has two parts.

Absolute Value—Algebraic Approach
Algebraic definition of absolute value
The absolute value of a number a is

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } < 0 \end{cases}$$

The algebraic definition takes into account the fact that the number a could be either positive or zero $(a \geq 0)$ or negative $(a < 0)$.

1. If the number a is positive or zero $(a \geq 0)$, the upper part of the definition applies. The upper part of the definition tells us that if the number enclosed in the absolute value bars is a nonnegative number, the absolute value of the number is the number itself.
2. The lower part of the definition tells us that if the number enclosed within the absolute value bars is a negative number, the absolute value of the number is the opposite of the number. The opposite of a negative number is a positive number.

Note: The definition says that the vertical absolute value lines may be eliminated only if we know whether the number inside is positive or negative.

Sample Set B

Use the algebraic definition of absolute value to find the following values.

Example:
$| \ 8 \ |$. The number enclosed within the absolute value bars is a nonnegative number, so the upper part of the definition applies. This part says that the absolute value of 8 is 8 itself.
$| \ 8 \ | = 8$

Example:
$| -3 |$. The number enclosed within absolute value bars is a negative number, so the lower part of the definition applies. This part says that the absolute value of -3 is the opposite of -3, which is $-(-3)$. By the definition of absolute value and the double-negative property,
$| -3 | = -(-3) = 3$

Practice Set B

Use the algebraic definition of absolute value to find the following values.
Exercise:

Problem: $| \ 7 \ |$

Solution:

7

Exercise:

Problem: $| \ 9 \ |$

Solution:

9

Exercise:

Problem: $\mid -12 \mid$

Solution:

12

Exercise:

Problem: $\mid -5 \mid$

Solution:

5

Exercise:

Problem: $- \mid 8 \mid$

Solution:

-8

Exercise:

Problem: $- \mid 1 \mid$

Solution:

-1

Exercise:

Problem: $- \mid -52 \mid$

Solution:

-52

Exercise:

Problem: $- \mid -31 \mid$

Solution:

-31

Exercises

Determine each of the values.

Exercise:

Problem: $\mid 5 \mid$

Solution:

5

Exercise:

Problem: $\mid 3 \mid$

Exercise:

Problem: $\mid 6 \mid$

Solution:

6

Exercise:

Problem: $\mid -9 \mid$

Exercise:

Problem: $\mid -1 \mid$

Solution:

1

Exercise:

Problem: $\mid -4 \mid$

Exercise:

Problem: $- \mid 3 \mid$

Solution:

-3

Exercise:

Problem: $- \mid 7 \mid$

Exercise:

Problem: $- \mid -14 \mid$

Solution:

-14

Exercise:

Problem: $\mid 0 \mid$

Exercise:

Problem: $\mid -26 \mid$

Solution:

26

Exercise:

Problem: $-\mid -26\mid$

Exercise:

Problem: $-(-\mid 4\mid)$

Solution:

4

Exercise:

Problem: $-(-\mid 2\mid)$

Exercise:

Problem: $-(-\mid -6\mid)$

Solution:

6

Exercise:

Problem: $-(-\mid -42\mid)$

Exercise:

Problem: $\mid 5\mid -\mid -2\mid$

Solution:

3

Exercise:

Problem: $\mid -2 \mid^3$

Exercise:

Problem: $\mid -(2 \cdot 3) \mid$

Solution:

6

Exercise:

Problem: $\mid -2 \mid - \mid -9 \mid$

Exercise:

Problem: $(\mid -6 \mid + \mid 4 \mid)^2$

Solution:

100

Exercise:

Problem: $(\mid -1 \mid - \mid 1 \mid)^3$

Exercise:

Problem: $(\mid 4 \mid + \mid -6 \mid)^2 - (\mid -2 \mid)^3$

Solution:

92

Exercise:

Problem: $-[|-10|-6]^2$

Exercise:

Problem: $-\left\{-[-\mid -4 \mid + \mid -3 \mid]^3\right\}^2$

Solution:

-1

Exercise:

Problem:

A Mission Control Officer at Cape Canaveral makes the statement "lift-off, *T* minus 50 seconds." How long is it before lift-off?

Exercise:

Problem:

Due to a slowdown in the industry, a Silicon Valley computer company finds itself in debt \$2,400,000. Use absolute value notation to describe this company's debt.

Solution:

$-\$ \mid -2{,}400{,}000 \mid$

Exercise:

Problem:

A particular machine is set correctly if upon action its meter reads 0. One particular machine has a meter reading of -1.6 upon action. How far is this machine off its correct setting?

Exercises for Review

Exercise:

Problem: ([link]) Find the sum: $\frac{9}{70} + \frac{5}{21} + \frac{8}{15}$.

Solution:

$\frac{9}{10}$

Exercise:

Problem: ([link]) Find the value of $\frac{\frac{3}{10} + \frac{4}{12}}{\frac{19}{20}}$.

Exercise:

Problem: ([link]) Convert $3.2\frac{3}{5}$ to a fraction.

Solution:

$3\frac{13}{50}$ or $\frac{163}{50}$

Exercise:

Problem:

([link]) The ratio of acid to water in a solution is $\frac{3}{8}$. How many mL of acid are there in a solution that contain 112 mL of water?

Exercise:

Problem: ([link]) Find the value of $-6 - (-8)$.

Solution:

2

Addition of Real Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add signed numbers. By the end of the module students should be able to add numbers with like signs and with unlike signs and be able to use the calculator for addition of signed numbers.

Section Overview

- Addition of Numbers with Like Signs
- Addition with Zero
- Addition of Numbers with Unlike Signs
- Calculators

Addition of Numbers with Like Signs

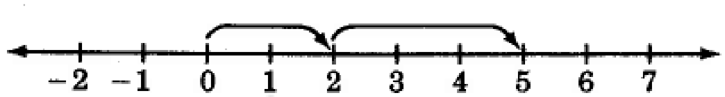
The addition of the *two positive numbers* 2 and 3 is performed on the number line as follows.

Begin at 0, the origin.

Since 2 is positive, move 2 units to the right.

Since 3 is positive, move 3 more units to the right.

We are now located at 5.



Thus, $2 + 3 = 5$.

Summarizing, we have

$(2 \text{ positive units}) + (3 \text{ positive units}) = (5 \text{ positive units})$

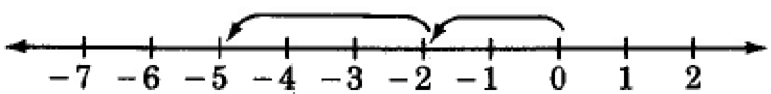
The addition of the *two negative numbers* -2 and -3 is performed on the number line as follows.

Begin at 0, the origin.

Since -2 is negative, move 2 units to the left.

Since -3 is negative, move 3 more units to the left.

We are now located at -5.



Thus, $(-2)+(-3)=-5$.

Summarizing, we have

$(2 \text{ negative units}) + (3 \text{ negative units}) = (5 \text{ negative units})$

Observing these two examples, we can suggest these relationships:

$(\text{positive number}) + (\text{positive number}) = (\text{positive number})$

$(\text{negative number}) + (\text{negative number}) = (\text{negative number})$

Adding Numbers with the Same Sign
Addition of numbers with like sign:
To add two real numbers that have the *same* sign, add the absolute values of the numbers and associate with the sum the common sign.

Sample Set A

Find the sums.

Example:
$3+7$
$$\left.\begin{array}{rcl}|3| & = & 3 \\ |7| & = & 7\end{array}\right\} \text{Add these absolute values.}$$
$3+7=10$
The common sign is “+.”
Thus, $3+7=+10$, or $3+7=10$.

Example:
$(-4)+(-9)$
$$\left.\begin{array}{rcl}|-4| & = & 4 \\ |-9| & = & 9\end{array}\right\} \text{Add these absolute values.}$$
$4+9=13$
The common sign is “$-$.“
Thus, $(-4)+(-9)=-13$.

Practice Set A

Find the sums.

Exercise:

Problem: $8+6$

Solution:

14

Exercise:

Problem: $41+11$

Solution:

52

Exercise:

Problem: $(-4)+(-8)$

Solution:

-12

Exercise:

Problem: $(-36)+(-9)$

Solution:

-45

Exercise:

Problem: $-14+(-20)$

Solution:

-34

Exercise:

Problem: $-\frac{2}{3}+\left(-\frac{5}{3}\right)$

Solution:

$-\frac{7}{3}$

Exercise:

Problem: $-2.8+(-4.6)$

Solution:

-7.4

Exercise:

Problem: $0+(-16)$

Solution:

-16

Addition With Zero

Addition with Zero
Notice that
$(0)+(\text{a positive number})=(\text{that same positive number}).$
$(0)+(\text{a negative number})=(\text{that same negative number}).$

The Additive Identity Is Zero
Since adding zero to a real number leaves that number unchanged, zero is called the **additive identity**.

Addition of Numbers with Unlike Signs

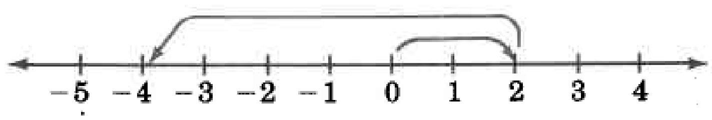
The addition $2+(-6)$, *two numbers with unlike signs*, can also be illustrated using the number line.

Begin at 0, the origin.

Since 2 is positive, move 2 units to the right.

Since -6 is negative, move, from 2, 6 units to the left.

We are now located at -4.



We can suggest a rule for adding two numbers that have *unlike signs* by noting that if the signs are disregarded, 4 can be obtained by subtracting 2 from 6. But 2 and 6 are precisely the absolute values of 2 and -6. Also, notice that the sign of the number with the larger absolute value is negative and that the sign of the resulting sum is negative.

Adding Numbers with Unlike Signs

Addition of numbers with unlike signs: To add two real numbers that have *unlike signs*, subtract the smaller absolute value from the larger absolute value and associate with this difference the sign of the number with the larger absolute value.

Sample Set B

Find the following sums.

Example:

$7+(-2)$

$|7|=7$ Larger absolute value. Sign is positive.

$|-2|=2$ Smaller absolute value.

Subtract absolute values: $7-2=5$.

Attach the proper sign: "+."

Thus, $7+(-2)=+5$ or $7+(-2)=5$.

Example:

$3+(-11)$

$|3| = 3$ $|-11| = 11$

Smaller absolute value. Larger absolute value. Sign is negative.

Subtract absolute values: $11 - 3 = 8$.
Attach the proper sign: "$-$."
Thus, $3 + (-11) = -8$.

Example:
The morning temperature on a winter's day in Lake Tahoe was -12 degrees. The afternoon temperature was 25 degrees warmer. What was the afternoon temperature?
We need to find $-12 + 25$.

$|-12| = 12$ $|25| = 25$

Smaller absolute value. Larger absolute value. Sign is positive.

Subtract absolute values: $25 - 12 = 16$.
Attach the proper sign: "+."
Thus, $-12 + 25 = 13$.

Practice Set B

Find the sums.

Exercise:

Problem: $4 + (-3)$

Solution:

1

Exercise:

Problem: $-3 + 5$

Solution:

2

Exercise:

Problem: $15 + (-18)$

Solution:

-3

Exercise:

Problem: $0 + (-6)$

Solution:

-6

Exercise:

Problem: $-26 + 12$

Solution:

-14

Exercise:

Problem: $35 + (-78)$

Solution:

-43

Exercise:

Problem: $15 + (-10)$

Solution:

5

Exercise:

Problem: $1.5 + (-2)$

Solution:

-0.5

Exercise:

Problem: $-8 + 0$

Solution:

-8

Exercise:

Problem: $0 + (0.57)$

Solution:

0.57

Exercise:

Problem: $-879 + 454$

Solution:

-425

Calculators

Calculators having the

$\boxed{+/-}$

key can be used for finding sums of signed numbers.

Sample Set C

Use a calculator to find the sum of -147 and 84.

		Display Reads	
Type	147	147	
Press	$\boxed{+/-}$	-147	This key changes the sign of a number. It is different than $-$.
Press	+	-147	
Type	84	84	
Press	=	-63	

Practice Set C

Use a calculator to find each sum.
Exercise:

Problem: $673 + (-721)$

Solution:

-48

Exercise:

Problem: $-8{,}261 + 2{,}206$

Solution:

-6,085

Exercise:

Problem: $-1{,}345.6 + (-6{,}648.1)$

Solution:

-7,993.7

Exercises

Find the sums in the following 27 problems. If possible, use a calculator to check each result.

Exercise:

Problem: $4 + 12$

Solution:

16

Exercise:

Problem: $8 + 6$

Exercise:

Problem: $(-3) + (-12)$

Solution:

-15

Exercise:

Problem: $(-6) + (-20)$

Exercise:

Problem: $10 + (-2)$

Solution:

8

Exercise:

Problem: $8 + (-15)$

Exercise:

Problem: $-16 + (-9)$

Solution:

-25

Exercise:

Problem: $-22 + (-1)$

Exercise:

Problem: $0 + (-12)$

Solution:

-12

Exercise:

Problem: $0 + (-4)$

Exercise:

Problem: $0 + (24)$

Solution:

24

Exercise:

Problem: $-6 + 1 + (-7)$

Exercise:

Problem: $-5 + (-12) + (-4)$

Solution:

-21

Exercise:

Problem: $-5 + 5$

Exercise:

Problem: $-7+7$

Solution:

0

Exercise:

Problem: $-14+14$

Exercise:

Problem: $4+(-4)$

Solution:

0

Exercise:

Problem: $9+(-9)$

Exercise:

Problem: $84+(-61)$

Solution:

23

Exercise:

Problem: $13+(-56)$

Exercise:

Problem: $452+(-124)$

Solution:

328

Exercise:

Problem: $636 + (-989)$

Exercise:

Problem: $1{,}811 + (-935)$

Solution:

876

Exercise:

Problem: $-373 + (-14)$

Exercise:

Problem: $-1{,}211 + (-44)$

Solution:

-1,255

Exercise:

Problem: $-47.03 + (-22.71)$

Exercise:

Problem: $-1.998 + (-4.086)$

Solution:

-6.084

Exercise:

Problem:

In order for a small business to break even on a project, it must have sales of $21,000. If the amount of sales was $15,000, by how much money did this company fall short?

Exercise:

Problem:

Suppose a person has $56 in his checking account. He deposits $100 into his checking account by using the automatic teller machine. He then writes a check for $84.50. If an error causes the deposit not to be listed into this person's account, what is this person's checking balance?

Solution:

-$28.50

Exercise:

Problem:

A person borrows $7 on Monday and then $12 on Tuesday. How much has this person borrowed?

Exercise:

Problem:

A person borrows $11 on Monday and then pays back $8 on Tuesday. How much does this person owe?

Solution:

$3.00

Exercises for Review

Exercise:

Problem: ([link]) Find the reciprocal of $8\frac{5}{6}$.

Exercise:

Problem: ([link]) Find the value of $\frac{5}{12}+\frac{7}{18}-\frac{1}{3}$.

Solution:

$\frac{17}{36}$

Exercise:

Problem: ([link]) Round 0.01628 to the nearest tenth.

Exercise:

Problem: ([link]) Convert 62% to a fraction.

Solution:

$\frac{62}{100}=\frac{31}{50}$

Exercise:

Problem: ([link]) Find the value of $\mid -12 \mid$.

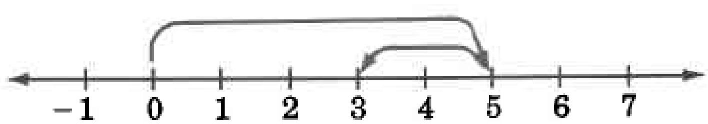
Subtraction of Real Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to subtract signed numbers. By the end of the module students should understand the definition of subtraction, be able to subtract signed numbers and be able to use a calculator to subtract signed numbers.

Section Overview

- Definition of Subtraction
- The Process of Subtraction
- Calculators

Definition of Subtraction

We know from experience with arithmetic that the subtraction $5 - 2$ produces 3, that is $5 - 2 = 3$. We can suggest a rule for subtracting signed numbers by illustrating this process on the number line.



Begin at 0, the origin.

Since 5 is positive, move 5 units to the right.

Then, move *2 units to the left* to get to 6. (This reminds us of addition with a negative number.)

From this illustration we can see that $5 - 2$ is the same as $5 + (-2)$. This leads us directly to the definition of subtraction.

Definition of Subtraction
If a and b are real numbers, $a - b$ is the same as $a + (-b)$, where $-b$ is the opposite of b.

The Process of Subtraction

From this definition, we suggest the following rule for subtracting signed numbers.

Subtraction of Signed Numbers
To perform the subtraction $a - b$, add the opposite of b to a, that is, change the sign of b and add.

Sample Set A

Perform the indicated subtractions.

Example:
$5 - 3 = 5 + (-3) = 2$

Example:
$4 - 9 = 4 + (-9) = -5$

Example:
$-4 - 6 = -4 + (-6) = -10$

Example:
$-3 - (-12) = -3 + 12 = 9$

Example:

$0-(-15)=0+15=15$

Example:
The high temperature today in Lake Tahoe was 26°F. The low temperature tonight is expected to be -7°F. How many degrees is the temperature expected to drop?
We need to find the difference between 26 and -7.
$26-(-7)=26+7=33$
Thus, the expected temperature drop is 33°F.

Example:

$$\begin{aligned} -6-(-5)-10 &= -6+5+(-10) \\ &= (-6+5)+(-10) \\ &= -1+(-10) \\ &= -11 \end{aligned}$$

Practice Set A

Perform the indicated subtractions.

Exercise:

Problem: $9-6$

Solution:

3

Exercise:

Problem: $6-9$

Solution:

-3

Exercise:

Problem: $0 - 7$

Solution:

-7

Exercise:

Problem: $1 - 14$

Solution:

-13

Exercise:

Problem: $-8 - 12$

Solution:

-20

Exercise:

Problem: $-21 - 6$

Solution:

-27

Exercise:

Problem: $-6-(-4)$

Solution:

-2

Exercise:

Problem: $8-(-10)$

Solution:

18

Exercise:

Problem: $1-(-12)$

Solution:

13

Exercise:

Problem: $86-(-32)$

Solution:

118

Exercise:

Problem: $0-16$

Solution:

-16

Exercise:

Problem: $0 - (-16)$

Solution:

16

Exercise:

Problem: $0 - (8)$

Solution:

-8

Exercise:

Problem: $5 - (-5)$

Solution:

10

Exercise:

Problem: $24 - [-(-24)]$

Solution:

0

Calculators

Calculators can be used for subtraction of signed numbers. The most efficient calculators are those with a

$\boxed{+/-}$

key.

Sample Set B

Use a calculator to find each difference.

Example:
$3{,}187 - 8{,}719$

Display Reads		
Type	3187	3187
Press	-	3187
Type	8719	8719
Press	=	-5532

Thus, $3{,}187 - 8{,}719 = -5{,}532$.

Example:
$-156 - (-211)$
Method A:

Display Reads		
Type	156	156
Press	$\boxed{+/-}$	-156
Type	-	-156
Press	211	211
Type	$\boxed{+/-}$	-211
Press	=	55

Thus, $-156-(-211)=55$.
Method B:
We manually change the subtraction to an addition and change the sign of the number to be subtracted.
$-156-(-211)$ becomes $-156+211$

Display Reads		
Type	156	156
Press	$\boxed{+/-}$	-156

Press	+	-156
Type	211	211
Press	=	55

Practice Set B

Use a calculator to find each difference.

Exercise:

Problem: $44 - 315$

Solution:

-271

Exercise:

Problem: $12.756 - 15.003$

Solution:

-2.247

Exercise:

Problem: $-31.89 - 44.17$

Solution:

-76.06

Exercise:

Problem: $-0.797 - (-0.615)$

Solution:

-0.182

Exercises

For the following 18 problems, perform each subtraction. Use a calculator to check each result.

Exercise:

Problem: $8 - 3$

Solution:

5

Exercise:

Problem: $12 - 7$

Exercise:

Problem: $5 - 6$

Solution:

-1

Exercise:

Problem: $14 - 30$

Exercise:

Problem: $-6-8$

Solution:

-14

Exercise:

Problem: $-1-12$

Exercise:

Problem: $-5-(-3)$

Solution:

-2

Exercise:

Problem: $-11-(-8)$

Exercise:

Problem: $0-6$

Solution:

-6

Exercise:

Problem: $0-15$

Exercise:

Problem: $0-(-7)$

Solution:

7

Exercise:

Problem: $0 - (-10)$

Exercise:

Problem: $67 - 38$

Solution:

29

Exercise:

Problem: $142 - 85$

Exercise:

Problem: $816 - 1140$

Solution:

-324

Exercise:

Problem: $105 - 421$

Exercise:

Problem: $-550 - (-121)$

Solution:

-429

Exercise:

Problem: $-15.016-(4.001)$

For the following 4 problems, perform the indicated operations.

Exercise:

Problem: $-26+7-52$

Solution:

-71

Exercise:

Problem: $-15-21-(-2)$

Exercise:

Problem: $-104-(-216)-(-52)$

Solution:

164

Exercise:

Problem: $-0.012-(-0.111)-(0.035)$

Exercise:

Problem:

When a particular machine is operating properly, its meter will read 34. If a broken bearing in the machine causes the meter reading to drop by 45 units, what is the meter reading?

Solution:

-11

Exercise:

Problem:

The low temperature today in Denver was $-4^\circ F$ and the high was $-42^\circ F$. What is the temperature difference?

Exercises for Review

Exercise:

Problem: ([link]) Convert $16.02\frac{1}{5}$ to a decimal.

Solution:

16.022

Exercise:

Problem: ([link]) Find 4.01 of 6.2.

Exercise:

Problem: ([link]) Convert $\frac{5}{16}$ to a percent.

Solution:

31.25%

Exercise:

Problem:

([link]) Use the distributive property to compute the product: $15 \cdot 82$.

Exercise:

Problem: ([link]) Find the sum: $16 + (-21)$.

Solution:

-5

Multiplication and Division of Real Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to multiply and divide signed numbers. By the end of the module students should be able to multiply and divide signed numbers and be able to multiply and divide signed numbers using a calculator.

Section Overview

- Multiplication of Signed Numbers
- Division of Signed Numbers
- Calculators

Multiplication of Signed Numbers

Let us consider first, the product of two positive numbers. Multiply: $3 \cdot 5$.

$3 \cdot 5$ means $5 + 5 + 5 = 15$

This suggests[footnote] that
In later mathematics courses, the word "suggests" turns into the word "proof." One example does not prove a claim. Mathematical proofs are constructed to validate a claim for all possible cases.

$(\text{positive number}) \cdot (\text{positive number}) = (\text{positive number})$

More briefly,

$(+)(+) = (+)$

Now consider the product of a positive number and a negative number. Multiply: $(3)(-5)$.

$(3)(-5)$ means $(-5) + (-5) + (-5) = -15$

This suggests that

$$(\text{positive number}) \cdot (\text{negative number}) = (\text{negative number})$$

More briefly,

$$(+)(-) = (-)$$

By the commutative property of multiplication, we get

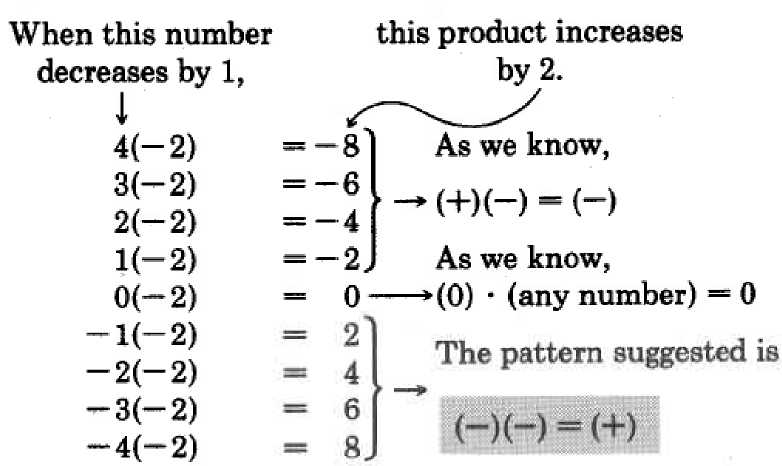
$$(\text{negative number}) \cdot (\text{positive number}) = (\text{negative number})$$

More briefly,

$$(-)(+) = (-)$$

The sign of the product of two negative numbers can be suggested after observing the following illustration.

Multiply -2 by, respectively, 4, 3, 2, 1, 0, -1, -2, -3, -4.



We have the following rules for multiplying signed numbers.

Rules for Multiplying Signed Numbers
Multiplying signed numbers:

1. To multiply two real numbers that have the *same sign*, multiply their absolute values. The product is positive.

$(+)(+) = (+)$
$(-)(-) = (+)$

2. To multiply two real numbers that have *opposite signs*, multiply their absolute values. The product is negative.
   $(+)(-) = (-)$
   $(-)(+) = (-)$

Sample Set A

Find the following products.

Example:
$8 \cdot 6$
$$\left.\begin{array}{rcl} |8| & = & 8 \\ |6| & = & 6 \end{array}\right\} \text{Multiply these absolute values.}$$
$8 \cdot 6 = 48$
Since the numbers have the same sign, the product is positive.
Thus, $8 \cdot 6 {=} {+}48$, or $8 \cdot 6 = 48$.

Example:
$(-8)(-6)$
$$\left.\begin{array}{rcl} |-8| & = & 8 \\ |-6| & = & 6 \end{array}\right\} \text{Multiply these absolute values.}$$
$8 \cdot 6 = 48$
Since the numbers have the same sign, the product is positive.
Thus, $(-8)(-6) {=} {+}48$, or $(-8)(-6) = 48$.

Example:
$(-4)(7)$

$$\left.\begin{array}{rcl} |-4| & = & 4 \\ |7| & = & 7 \end{array}\right\}$$ Multiply these absolute values.
$4 \cdot 7 = 28$
Since the numbers have opposite signs, the product is negative.
Thus, $(-4)(7) = -28$.

Example:
$6(-3)$
$$\left.\begin{array}{rcl} |6| & = & 6 \\ |-3| & = & 3 \end{array}\right\}$$ Multiply these absolute values.
$6 \cdot 3 = 18$
Since the numbers have opposite signs, the product is negative.
Thus, $6(-3) = -18$.

Practice Set A

Find the following products.
Exercise:

Problem: $3(-8)$

Solution:

-24

Exercise:

Problem: $4(16)$

Solution:

64

Exercise:

Problem: $(-6)(-5)$

Solution:

30

Exercise:

Problem: $(-7)(-2)$

Solution:

14

Exercise:

Problem: $(-1)(4)$

Solution:

-4

Exercise:

Problem: $(-7)7$

Solution:

-49

Division of Signed Numbers

To determine the signs in a division problem, recall that

$\frac{12}{3} = 4$ since $12 = 3 \cdot 4$

This suggests that

$\frac{(+)}{(+)} = (+)$
$\frac{(+)}{(+)} = (+)$ since $(+) = (+)(+)$

What is $\frac{12}{-3}$?

$-12 = (-3)(-4)$ suggests that $\frac{12}{-3} = -4$. That is,

$\frac{(+)}{(-)} = (-)$
$(+) = (-)(-)$ suggests that $\frac{(+)}{(-)} = (-)$

What is $\frac{-12}{3}$?

$-12 = (3)(-4)$ suggests that $\frac{-12}{3} = -4$. That is,

$\frac{(-)}{(+)} = (-)$
$(-) = (+)(-)$ suggests that $\frac{(-)}{(+)} = (-)$

What is $\frac{-12}{-3}$?

$-12 = (-3)(4)$ suggests that $\frac{-12}{-3} = 4$. That is,

$\frac{(-)}{(-)} = (+)$
$(-) = (-)(+)$ suggests that $\frac{(-)}{(-)} = (+)$

We have the following rules for dividing signed numbers.

Rules for Dividing Signed Numbers
Dividing signed numbers:

1. To divide two real numbers that have the *same sign*, divide their absolute values. The quotient is positive.

$\frac{(+)}{(+)} = (+)$ $\frac{(-)}{(-)} = (+)$

2. To divide two real numbers that have *opposite signs*, divide their absolute values. The quotient is negative.
   $\frac{(-)}{(+)} = (-)$ $\frac{(+)}{(-)} = (-)$

Sample Set B

Find the following quotients.

Example:
$\frac{-10}{2}$
$\left.\begin{array}{rcr} |-10| & = & 10 \\ |2| & = & 2 \end{array}\right\}$ Divide these absolute values.
$\frac{10}{2} = 5$
Since the numbers have opposite signs, the quotient is negative.
Thus $\frac{-10}{2} = -5$.

Example:
$\frac{-35}{-7}$
$\left.\begin{array}{rcr} |-35| & = & 35 \\ |-7| & = & 7 \end{array}\right\}$ Divide these absolute values.
$\frac{35}{7} = 5$
Since the numbers have the same signs, the quotient is positive.
Thus, $\frac{-35}{-7} = 5$.

Example:
$\frac{18}{-9}$

$$\left.\begin{array}{rcr} |18| & = & 18 \\ |-9| & = & 9 \end{array}\right\} \text{Divide these absolute values.}$$

$\frac{18}{9} = 2$

Since the numbers have opposite signs, the quotient is negative.

Thus, $\frac{18}{-9} = 2$.

Practice Set B

Find the following quotients.

Exercise:

Problem: $\frac{-24}{-6}$

Solution:

4

Exercise:

Problem: $\frac{30}{-5}$

Solution:

-6

Exercise:

Problem: $\frac{-54}{27}$

Solution:

-2

Exercise:

Problem: $\frac{51}{17}$

Solution:

3

Sample Set C

Example:
Find the value of $\frac{-6(4-7)-2(8-9)}{-(4+1)+1}$.
Using the order of operations and what we know about signed numbers, we get,

$$\begin{aligned}\frac{-6(4-7)-2(8-9)}{-(4+1)+1} &= \frac{-6(-3)-2(-1)}{-(5)+1}\\ &= \frac{18+2}{-5+1}\\ &= \text{mfrac}\\ &= -5\end{aligned}$$

Practice Set C

Exercise:

Problem: Find the value of $\frac{-5(2-6)-4(-8-1)}{2(3-10)-9(-2)}$.

Solution:

14

Calculators

Calculators with the

$\boxed{+/-}$

key can be used for multiplying and dividing signed numbers.

Sample Set D

Use a calculator to find each quotient or product.

Example:
$(-186) \cdot (-43)$
Since this product involves a $(\text{negative}) \cdot (\text{negative})$, we know the result should be a positive number. We'll illustrate this on the calculator.

		Display Reads
Type	186	186
Press	$\boxed{+/-}$	-186
Press	×	-186
Type	43	43

Press	$\boxed{+/-}$	-43
Press	=	7998

Thus, $(-186) \cdot (-43) = 7{,}998$.

Example:
$\frac{158.64}{-54.3}$. Round to one decimal place.

		Display Reads
Type	158.64	158.64
Press	÷	158.64
Type	54.3	54.3
Press	$\boxed{+/-}$	-54.3
Press	=	-2.921546961

Rounding to one decimal place we get -2.9.

Practice Set D

Use a calculator to find each value.
Exercise:

Problem: $(-51.3) \cdot (-21.6)$

Solution:

1,108.08

Exercise:

Problem: $-2.5746 \div -2.1$

Solution:

1.226

Exercise:

Problem: $(0.006) \cdot (-0.241)$. Round to three decimal places.

Solution:

-0.001

Exercises

Find the value of each of the following. Use a calculator to check each result.
Exercise:

Problem: $(-2)(-8)$

Solution:

16

Exercise:

Problem: $(-3)(-9)$

Exercise:

Problem: $(-4)(-8)$

Solution:

32

Exercise:

Problem: $(-5)(-2)$

Exercise:

Problem: $(3)(-12)$

Solution:

-36

Exercise:

Problem: $(4)(-18)$

Exercise:

Problem: $(10)(-6)$

Solution:

-60

Exercise:

Problem: $(-6)(4)$

Exercise:

Problem: $(-2)(6)$

Solution:

-12

Exercise:

Problem: $(-8)(7)$

Exercise:

Problem: $\frac{21}{7}$

Solution:

3

Exercise:

Problem: $\frac{42}{6}$

Exercise:

Problem: $\frac{-39}{3}$

Solution:

-13

Exercise:

Problem: $\frac{-20}{10}$

Exercise:

Problem: $\frac{-45}{-5}$

Solution:

9

Exercise:

Problem: $\frac{-16}{-8}$

Exercise:

Problem: $\frac{25}{-5}$

Solution:

-5

Exercise:

Problem: $\frac{36}{-4}$

Exercise:

Problem: $8-(-3)$

Solution:

11

Exercise:

Problem: $14-(-20)$

Exercise:

Problem: $20 - (-8)$

Solution:

28

Exercise:

Problem: $-4 - (-1)$

Exercise:

Problem: $0 - 4$

Solution:

-4

Exercise:

Problem: $0 - (-1)$

Exercise:

Problem: $-6 + 1 - 7$

Solution:

-12

Exercise:

Problem: $15 - 12 - 20$

Exercise:

Problem: $1 - 6 - 7 + 8$

Solution:

-4

Exercise:

Problem: $2 + 7 - 10 + 2$

Exercise:

Problem: $3(4 - 6)$

Solution:

-6

Exercise:

Problem: $8(5 - 12)$

Exercise:

Problem: $-3(1 - 6)$

Solution:

15

Exercise:

Problem: $-8(4 - 12) + 2$

Exercise:

Problem: $-4(1 - 8) + 3(10 - 3)$

Solution:

49

Exercise:

Problem: $-9(0-2)+4(8-9)+0(-3)$

Exercise:

Problem: $6(-2-9)-6(2+9)+4(-1-1)$

Solution:

-140

Exercise:

Problem: $\frac{3(4+1)-2(5)}{-2}$

Exercise:

Problem: $\frac{4(8+1)-3(-2)}{-4-2}$

Solution:

-7

Exercise:

Problem: $\frac{-1(3+2)+5}{-1}$

Exercise:

Problem: $\frac{-3(4-2)+(-3)(-6)}{-4}$

Solution:

-3

Exercise:

Problem: $-1(4+2)$

Exercise:

Problem: $-1(6-1)$

Solution:

-5

Exercise:

Problem: $-(8+21)$

Exercise:

Problem: $-(8-21)$

Solution:

13

Exercises for Review

Exercise:

Problem:

([link]) Use the order of operations to simplify $\left(5^2+3^2+2\right)\div 2^2$.

Exercise:

Problem: ([link]) Find $\frac{3}{8}$ of $\frac{32}{9}$.

Solution:

$\frac{4}{3} = 1\frac{1}{3}$

Exercise:

Problem:

([link]) Write this number in decimal form using digits: “fifty-two three-thousandths”

Exercise:

Problem:

([link]) The ratio of chlorine to water in a solution is 2 to 7. How many mL of water are in a solution that contains 15 mL of chlorine?

Solution:

$52\frac{1}{2}$

Exercise:

Problem: ([link]) Perform the subtraction $-8-(-20)$

Algebraic Expressions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses algebraic expressions. By the end of the module students should be able to recognize an algebraic expression, be able to distinguish between terms and factors, understand the meaning and function of coefficients and be able to perform numerical evaluation.

Section Overview

- Algebraic Expressions
- Terms and Factors
- Coefficients
- Numerical Evaluation

Algebraic Expressions

Numerical Expression
In arithmetic, a **numerical expression** results when numbers are connected by arithmetic operation signs (+, -, · , ÷). For example, $8+5$, $4-9$, $3 \cdot 8$, and $9 \div 7$ are numerical expressions.

Algebraic Expression
In algebra, letters are used to represent numbers, and an **algebraic expression** results when an arithmetic operation sign associates a letter with a number or a letter with a letter. For example, $x+8$, $4-y$, $3 \cdot x$, $x \div 7$, and $x \cdot y$ are algebraic expressions.

Expressions
Numerical expressions and algebraic expressions are often referred to simply as **expressions**.

Terms and Factors

In algebra, it is extremely important to be able to distinguish between terms and factors.

Distinction Between Terms and Factors
Terms are parts of *sums* and are therefore connected by + signs.
Factors are parts of *products* and are therefore separated by · signs.

Note: While making the distinction between sums and products, we must remember that subtraction and division are functions of these operations.

1. In some expressions it will appear that terms are separated by minus signs. We must keep in mind that subtraction is addition of the opposite, that is,
   $x-y=x+(-y)$
2. In some expressions it will appear that factors are separated by division signs. We must keep in mind that
   $\frac{x}{y}=\frac{x}{1} \cdot \frac{1}{y}=x \cdot \frac{1}{y}$

Sample Set A

State the number of terms in each expression and name them.

Example:
$x + 4$. In this expression, *x* and 4 are connected by a "+" sign. Therefore, they are terms. This expression consists of two terms.

Example:
$y - 8$. The expression $y - 8$ can be expressed as $y + (-8)$. We can now see that this expression consists of the two terms y and -8.
Rather than rewriting the expression when a subtraction occurs, we can identify terms more quickly by associating the + or **-** sign with the individual quantity.

Example:
$a + 7 - b - m$. Associating the sign with the individual quantities, we see that this expression consists of the four terms a, 7, $-b$, $-m$.

Example:
$5m - 8n$. This expression consists of the two terms, $5m$ and $-8n$. Notice that the term $5m$ is composed of the two factors 5 and m. The term $-8n$ is composed of the two factors -8 and n.

Example:
$3x$. This expression consists of one term. Notice that $3x$ can be expressed as $3x + 0$ or $3x \cdot 1$ (indicating the connecting signs of arithmetic). Note that no operation sign is necessary for multiplication.

Practice Set A

Specify the terms in each expression.

Exercise:

Problem: $x + 7$

Solution:

x, 7

Exercise:

Problem: $3m - 6n$

Solution:

$3m - 6n$

Exercise:

Problem: $5y$

Solution:

$5y$

Exercise:

Problem: $a + 2b - c$

Solution:

$a, 2b, -c$

Exercise:

Problem: $-3x - 5$

Solution:

$-3x, -5$

Coefficients

We know that multiplication is a description of repeated addition. For example,
$5 \cdot 7$ describes $7 + 7 + 7 + 7 + 7$

Suppose some quantity is represented by the letter x. The multiplication 5x describes $x + x + x + x + x$. It is now easy to see that $5x$ specifies 5 of the quantities represented by x. In the expression $5x$, 5 is called the **numerical coefficient**, or more simply, the **coefficient** of x.

Coefficient
The **coefficient** of a quantity records how many of that quantity there are.

Since constants alone do not record the number of some quantity, they are not usually considered as numerical coefficients. For example, in the expression $7x + 2y - 8z + 12$, the coefficient of

$7x$ is 7. (There are 7 *x*'s.)
$2y$ is 2. (There are 2 *y* 's.)
$-8z$ is -8. (There are $-8z$'s.)

The constant 12 is not considered a numerical coefficient.

$1x = x$
When the numerical coefficient of a variable is 1, we write only the variable and not the coefficient. For example, we write x rather than $1x$. It is clear just by looking at x that there is only one.

Numerical Evaluation

We know that a variable represents an unknown quantity. Therefore, any expression that contains a variable represents an unknown quantity. For example, if the value of x is unknown, then the value of $3x + 5$ is unknown. The value of $3x + 5$ depends on the value of x.

Numerical Evaluation
Numerical evaluation is the process of determining the numerical value of an algebraic expression by replacing the variables in the expression with specified numbers.

Sample Set B

Find the value of each expression.

Example:
$2x + 7y$, if $x = -4$ and $y = 2$
Replace *x* with –4 and *y* with 2.

$$\begin{aligned} 2x + 7y &= 2(-4) + 7(2) \\ &= -8 + 14 \\ &= 6 \end{aligned}$$

Thus, when $x = -4$ and $y = 2$, $2x + 7y = 6$.

Example:
$\frac{5a}{b} + \frac{8b}{12}$, if $a = 6$ and $b = -3$.
Replace *a* with 6 and *b* with –3.

$$\begin{aligned} \frac{5a}{b} + \frac{8b}{12} &= \frac{5(6)}{-3} + \frac{8(-3)}{12} \\ &= \text{mfrac} + \text{mfrac} \\ &= -10 + (-2) \\ &= -12 \end{aligned}$$

Thus, when *a* = 6 and *b* = –3, $\frac{5a}{b} + \frac{8b}{12} = -12$.

Example:
$6(2a - 15b)$, if $a = -5$ and $b = -1$
Replace *a* with –5 and *b* with –1.

$$\begin{aligned} 6(2a - 15b) &= 6(2(-5) - 15(-1)) \\ &= 6(-10 + 15) \\ &= 6(5) \\ &= 30 \end{aligned}$$

Thus, when $a = –5$ and $b = –1$, $6(2a - 15b) = 30$.

Example:
$3x^2 - 2x + 1$, if $x = 4$
Replace *x* with 4.

$$\begin{aligned} 3x^2 - 2x + 1 &= 3(4)^2 - 2\left(4\right) + 1 \\ &= 3 \cdot 16 - 2(4) + 1 \\ &= 48 - 8 + 1 \\ &= 41 \end{aligned}$$

Thus, when $x = 4$, $3x^2 - 2x + 1 = 41$.

Example:
$-x^2 - 4$, if $x = 3$
Replace *x* with 3.
$$\begin{array}{rcll} -x^2 - 4 & = & -\frac{3}{2} - 4 & \text{Be careful to square only the 3. The exponent 2 is connected } \textit{only} \text{ to 3, not -3} \\ & = & -9 - 4 & \\ & = & -13 & \end{array}$$

Example:
$(-x)^2 - 4$, if $x = 3$.
Replace *x* with 3.
$$\begin{array}{rcll} (-x)^2 - 4 & = & (-3)^2 - 4 & \text{The exponent is connected to -3, not 3 as in problem 5 above.} \\ & = & 9 - 4 & \\ & = & -5 & \end{array}$$
The exponent is connected to –3, not 3 as in the problem above.

Practice Set B

Find the value of each expression.
Exercise:

Problem: $9m - 2n$, if $m = -2$ and $n = 5$

Solution:

-28
Exercise:

Problem: $-3x - 5y + 2z$, if $x = -4$, $y = 3$, $z = 0$

Solution:

-3
Exercise:

Problem: $\frac{10a}{3\text{b}} + \frac{4\text{b}}{2}$, if $a = -6$, and $b = 2$

Solution:

-6
Exercise:

Problem: $8(3m - 5n)$, if $m = -4$ and $n = -5$

Solution:

104

Exercise:

Problem: $3[-40-2(4a-3b)]$, if $a=-6$ and $b=0$

Solution:

24

Exercise:

Problem: $5y^2+6y-11$, if $y=-1$

Solution:

-12

Exercise:

Problem: $-x^2+2x+7$, if $x=4$

Solution:

-1

Exercise:

Problem: $(-x)^2+2x+7$, if $x=4$

Solution:

31

Exercises

Exercise:

Problem: In an algebraic expression, terms are separated by signs and factors are separated by signs.

Solution:

Addition; multiplication

For the following 8 problems, specify each term.

Exercise:

Problem: $3m+7n$

Exercise:

Problem: $5x+18y$

Solution:

$5x, 18y$

Exercise:

Problem: $4a - 6b + c$

Exercise:

Problem: $8s + 2r - 7t$

Solution:

$8s, 2r, -7t$

Exercise:

Problem: $m - 3n - 4a + 7b$

Exercise:

Problem: $7a - 2b - 3c - 4d$

Solution:

$7a, -2b, -3c, -4d$

Exercise:

Problem: $-6a - 5b$

Exercise:

Problem: $-x - y$

Solution:

$-x, -y$

Exercise:

Problem: What is the function of a numerical coefficient?

Exercise:

Problem: Write $1m$ in a simpler way.

Solution:

m

Exercise:

Problem: Write $1s$ in a simpler way.

Exercise:

Problem: In the expression $5a$, how many a's are indicated?

Solution:

5

Exercise:

Problem: In the expression –7*c*, how many *c*’s are indicated?

Find the value of each expression.

Exercise:

Problem: $2m - 6n$, if $m = -3$ and $n = 4$

Solution:

-30

Exercise:

Problem: $5a + 6b$, if $a = -6$ and $b = 5$

Exercise:

Problem: $2x - 3y + 4z$, if $x = 1$, $y = -1$, and $z = -2$

Solution:

-3

Exercise:

Problem: $9a + 6b - 8x + 4y$, if $a = -2$, $b = -1$, $x = -2$, and $y = 0$

Exercise:

Problem: $\frac{8x}{3y} + \frac{18y}{2x}$, if $x = 9$ and $y = -2$

Solution:

-14

Exercise:

Problem: $\frac{-3m}{2n} - \frac{-6n}{m}$, if $m = -6$ and $n = 3$

Exercise:

Problem: $4(3r + 2s)$, if $r = 4$ and $s = 1$

Solution:

56

Exercise:

Problem: $3(9a-6b)$, if $a=-1$ and $b=-2$

Exercise:

Problem: $-8(5m+8n)$, if $m=0$ and $n=-1$

Solution:

64

Exercise:

Problem: $-2(-6x+y-2z)$, if $x=1$, $y=1$, and $z=2$

Exercise:

Problem: $-(10x-2y+5z)$ if $x=2$, $y=8$, and $z=-1$

Solution:

1

Exercise:

Problem: $-(a-3b+2c-d)$, if $a=-5$, $b=2$, $c=0$, and $d=-1$

Exercise:

Problem: $3[16-3(a+3b)]$, if $a=3$ and $b=-2$

Solution:

75

Exercise:

Problem: $-2[5a+2b(b-6)]$, if $a=-2$ and $b=3$

Exercise:

Problem: $-\{6x+3y[-2(x+4y)]\}$, if $x=0$ and $y=1$

Solution:

24

Exercise:

Problem: $-2\{19-6[4-2(a-b-7)]\}$, if $a=10$ and $b=3$

Exercise:

Problem: x^2+3x-1, if $x=5$

Solution:

39

Exercise:

Problem: $m^2 - 2m + 6$, if $m = 3$

Exercise:

Problem: $6a^2 + 2a - 15$, if $a = -2$

Solution:

5

Exercise:

Problem: $5s^2 + 6s + 10$, if $x = -1$

Exercise:

Problem: $16x^2 + 8x - 7$, if $x = 0$

Solution:

-7

Exercise:

Problem: $-8y^2 + 6y + 11$, if $y = 0$

Exercise:

Problem: $(y - 6)^2 + 3(y - 5) + 4$, if $y = 5$

Solution:

5

Exercise:

Problem: $(x + 8)^2 + 4(x + 9) + 1$, if $x = -6$

Exercises for Review

Exercise:

Problem: ([link]) Perform the addition: $5\frac{3}{8} + 2\frac{1}{6}$.

Solution:

$\frac{181}{24} = 7\frac{13}{24}$

Exercise:

Problem: ([link]) Arrange the numbers in order from smallest to largest: $\frac{11}{32}, \frac{15}{48}, \text{and} \frac{7}{16}$

Exercise:

Problem: ([link]) Find the value of $\left(\frac{2}{3}\right)^2 + \frac{8}{27}$

Solution:

$\frac{20}{27}$

Exercise:

Problem: ([link]) Write the proportion in fractional form: "9 is to 8 as *x* is to 7."

Exercise:

Problem: ([link]) Find the value of $-3(2-6)-12$

Solution:

0

Translations
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to translate word to mathematical symbols. By the end of the module students should be able to translate phrases and statements to mathematical expressions and equations.

Section Overview

- Translating Words to Symbols

Translating Words to Symbols

Practical problems seldom, if ever, come in equation form. The job of the problem solver is to translate the problem from phrases and statements into mathematical expressions and equations, and then to solve the equations.

As problem solvers, our job is made simpler if we are able to translate verbal phrases to mathematical expressions and if we follow the five-step method of solving applied problems. To help us translate from words to symbols, we can use the following Mathematics Dictionary.

MATHEMATICS DICTIONARY	
Word or Phrase	Mathematical Operation
Sum, sum of, added to, increased by, more than, and, plus	+
Difference, minus, subtracted from, decreased by, less, less than	-

Product, the product of, of, multiplied by, times, per	$\cdot$
Quotient, divided by, ratio, per	$\div$
Equals, is equal to, is, the result is, becomes	$=$
A number, an unknown quantity, an unknown, a quantity	x (or any symbol)

Sample Set A

Translate each phrase or sentence into a mathematical expression or equation.

Example:
$\underbrace{\text{Nine}}_{9}\underbrace{\text{more than}}_{+}\underbrace{\text{some number}}_{x}.$
Translation: $9 + x$.

Example:
$\underbrace{\text{Eighteen}}_{18}\underbrace{\text{minus}}_{-}\underbrace{\text{a number}}_{x}.$
Translation: $18 - x$.

Example:
$\underbrace{\text{A quantity}}_{y}\underbrace{\text{less}}_{-}\underbrace{\text{five}}_{5}.$
Translation: $y - 5$.

Example:

$$\underset{4}{\text{Four}}\ \underset{\cdot}{\text{times}}\ \underset{x}{\text{a number}}\ \underset{=}{\text{is}}\ \underset{16}{\text{sixteen.}}$$

Translation: $4x = 16$.

Example:

$$\underset{\frac{1}{5}}{\text{One fifth}}\ \underset{\cdot}{\text{of}}\ \underset{n}{\text{a number}}\ \underset{=}{\text{is}}\ \underset{30}{\text{thirty.}}$$

Translation: $\frac{1}{5}n = 30$, or $\frac{n}{5} = 30$.

Example:

$$\underset{5}{\text{Five}}\ \underset{\cdot}{\text{times}}\ \underset{x}{\text{a number}}\ \underset{=}{\text{is}}\ \underset{2}{\text{two}}\ \underset{+}{\text{more than}}\ \underset{2\cdot}{\text{twice}}\ \underset{x}{\text{the number.}}$$

Translation: $5x = 2 + 2x$.

Practice Set A

Translate each phrase or sentence into a mathematical expression or equation.

Exercise:

Problem: Twelve more than a number.

Solution:

$12 + x$

Exercise:

Problem: Eight minus a number.

Solution:

$8 - x$

Exercise:

Problem: An unknown quantity less fourteen.

Solution:

$x - 14$

Exercise:

Problem: Six times a number is fifty-four.

Solution:

$6x = 54$

Exercise:

Problem: Two ninths of a number is eleven.

Solution:

$$\frac{2}{9}x = 11$$

Exercise:

Problem:

Three more than seven times a number is nine more than five times the number.

Solution:

$3 + 7x = 9 + 5x$

Exercise:

Problem:

Twice a number less eight is equal to one more than three times the number.

Solution:

$2x - 8 = 3x + 1$ or $2x - 8 = 1 + 3x$

Sample Set B

Example:

Sometimes the structure of the sentence indicates the use of grouping symbols. We'll be alert for *commas*. They set off terms.

$$\underbrace{\text{A number}}_{(x}\ \underbrace{\text{divided by}}_{\div}\ \underbrace{\text{four,}}_{4)}\ \underbrace{\text{minus}}_{-}\ \underbrace{\text{six,}}_{6}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{twelve}}_{12}$$

Translation: $\frac{x}{4} - 6 = 12$.

Example:

Some phrases and sentences do not translate directly. We must be careful to read them properly. The word *from* often appears in such phrases and sentences. The word **from** means "a point of departure for motion." The following translation will illustrate this use.

Translation: $x - 20$.

The word *from* indicated the motion (subtraction) is to begin at the point of "some number."

Example:
Ten less than some number. Notice that *less than* can be replaced by *from*.
Ten from some number.
Translation: $x - 10$.

Practice Set B

Translate each phrase or sentence into a mathematical expression or equation.

Exercise:

Problem: A number divided by eight, plus seven, is fifty.

Solution:

$$\frac{x}{8} + 7 = 50$$

Exercise:

Problem:

A number divided by three, minus the same number multiplied by six, is one more than the number.

Solution:

$$\frac{2}{3} - 6x = x + 1$$

Exercise:

Problem: Nine from some number is four.

Solution:

$x - 9 = 4$

Exercise:

Problem: Five less than some quantity is eight.

Solution:

$x - 5 = 8$

Exercises

Translate each phrase or sentence to a mathematical expression or equation.

Exercise:

Problem: A quantity less twelve.

Solution:

$x - 12$

Exercise:

Problem: Six more than an unknown number.

Exercise:

Problem: A number minus four.

Solution:

$x - 4$

Exercise:

Problem: A number plus seven.

Exercise:

Problem: A number increased by one.

Solution:

$x + 1$

Exercise:

Problem: A number decreased by ten.

Exercise:

Problem: Negative seven added to some number.

Solution:

$-7 + x$

Exercise:

Problem: Negative nine added to a number.

Exercise:

Problem: A number plus the opposite of six.

Solution:

$x + (-6)$

Exercise:

Problem: A number minus the opposite of five.

Exercise:

Problem: A number minus the opposite of negative one.

Solution:

$x - [-(-1)]$

Exercise:

Problem: A number minus the opposite of negative twelve.

Exercise:

Problem: Eleven added to three times a number.

Solution:

$3x + 11$

Exercise:

Problem: Six plus five times an unknown number.

Exercise:

Problem: Twice a number minus seven equals four.

Solution:

$2x - 7 = 4$

Exercise:

Problem: Ten times a quantity increased by two is nine.

Exercise:

Problem:

When fourteen is added to two times a number the result is six.

Solution:

$14 + 2x = 6$

Exercise:

Problem: Four times a number minus twenty-nine is eleven.

Exercise:

Problem: Three fifths of a number plus eight is fifty.

Solution:

$$\frac{3}{5}x + 8 = 50$$

Exercise:

Problem: Two ninths of a number plus one fifth is forty-one.

Exercise:

Problem:

When four thirds of a number is increased by twelve, the result is five.

Solution:

$$\frac{4}{3}x + 12 = 5$$

Exercise:

Problem:

When seven times a number is decreased by two times the number, the result is negative one.

Exercise:

Problem:

When eight times a number is increased by five, the result is equal to the original number plus twenty-six.

Solution:

$8x + 5 = x + 26$

Exercise:

Problem:

Five more than some number is three more than four times the number.

Exercise:

Problem:

When a number divided by six is increased by nine, the result is one.

Solution:

$\frac{x}{6} + 9 = 1$

Exercise:

Problem: A number is equal to itself minus three times itself.

Exercise:

Problem: A number divided by seven, plus two, is seventeen.

Solution:

$$\frac{x}{7} + 2 = 17$$

Exercise:

Problem:

A number divided by nine, minus five times the number, is equal to one more than the number.

Exercise:

Problem: When two is subtracted from some number, the result is ten.

Solution:

$$x - 2 = 10$$

Exercise:

Problem:

When four is subtracted from some number, the result is thirty-one.

Exercise:

Problem:

Three less than some number is equal to twice the number minus six.

Solution:

$$x - 3 = 2x - 6$$

Exercise:

Problem:

Thirteen less than some number is equal to three times the number added to eight.

Exercise:

Problem:

When twelve is subtracted from five times some number, the result is two less than the original number.

Solution:

$5x - 12 = x - 2$

Exercise:

Problem:

When one is subtracted from three times a number, the result is eight less than six times the original number.

Exercise:

Problem:

When a number is subtracted from six, the result is four more than the original number.

Solution:

$6 - x = x + 4$

Exercise:

Problem:

When a number is subtracted from twenty-four, the result is six less than twice the number.

Exercise:

Problem:

A number is subtracted from nine. This result is then increased by one. The result is eight more than three times the number.

Solution:

$9 - x + 1 = 3x + 8$

Exercise:

Problem:

Five times a number is increased by two. This result is then decreased by three times the number. The result is three more than three times the number.

Exercise:

Problem:

Twice a number is decreased by seven. This result is decreased by four times the number. The result is negative the original number, minus six.

Solution:

$2x - 7 - 4x = -x - 6$

Exercise:

Problem:

Fifteen times a number is decreased by fifteen. This result is then increased by two times the number. The result is negative five times the original number minus the opposite of ten.

Exercises for Review

Exercise:

Problem: ([link]) $\frac{8}{9}$ of what number is $\frac{2}{3}$?

Solution:

$$\frac{3}{4}$$

Exercise:

Problem: ([link]) Find the value of $\frac{21}{40} + \frac{17}{30}$.

Exercise:

Problem: ([link]) Find the value of $3\frac{1}{12} + 4\frac{1}{3} + 1\frac{1}{4}$.

Solution:

$$8\frac{2}{3}$$

Exercise:

Problem: ([link]) Convert $6.11\frac{1}{5}$ to a fraction.

Exercise:

Problem: ([link]) Solve the equation $\frac{3x}{4} + 1 = -5$.

Solution:

$$x = -8$$

Simplifying Algebraic Expressions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to solve algebraic problems. By the end of the module students should be more familiar with the five-step method for solving applied problems and be able to use the five-step method to solve number problems and geometry problems.

Section Overview

- The Five-Step Method
- Number Problems
- Geometry Problems

The Five Step Method

We are now in a position to solve some applied problems using algebraic methods. The problems we shall solve are intended as logic developers. Although they may not seem to reflect real situations, they do serve as a basis for solving more complex, real situation, applied problems. To solve problems algebraically, we will use the five-step method.

Strategy for Reading Word Problems
When solving mathematical word problems, you may wish to apply the following "**reading strategy**." Read the problem quickly to get a feel for the situation. Do not pay close attention to details. At the first reading, too much attention to details may be overwhelming and lead to confusion and discouragement. After the first, brief reading, read the problem carefully in *phrases*. Reading phrases introduces information more slowly and allows us to absorb and put together important information. We can look for the unknown quantity by reading one phrase at a time.
Five-Step Method for Solving Word Problems

1. Let x (or some other letter) represent the unknown quantity.
2. Translate the words to mathematical symbols and form an equation. Draw a picture if possible.
3. Solve the equation.
4. Check the solution by substituting the result into the original statement, not equation, of the problem.
5. Write a conclusion.

If it has been your experience that word problems are difficult, then follow the five-step method carefully. Most people have trouble with word problems for two reasons:

1. They are not able to translate the words to mathematical symbols. (See [link].)
2. They neglect step 1. After working through the problem phrase by phrase, to become familiar with the situation,

INTRODUCE A VARIABLE

Number Problems

Sample Set A

Example:
What number decreased by six is five?

Let n represent the unknown number.
Translate the words to mathematical symbols and construct an equation. Read phrases.

What number:	n	
decreased by:	$-$	
six:	6	$n-6=5$
is:	$=$	
five:	5	

Solve this equation.

$n-6=5$ Add 6 to *both* sides.

$n-6+6=5+6$

$n=11$

Check the result.

When 11 is decreased by 6, the result is $11-6$, which is equal to 5. The solution checks.

The number is 11.

Example:

When three times a number is increased by four, the result is eight more than five times the number.

Let $x=$ the unknown number.

Translate the phrases to mathematical symbols and construct an equation.

When three times a number:	$3x$	
is increased by:	$+$	
four:	4	
the result is:	$=$	$3x+4=5x+8$
eight:	8	
more than:	$+$	
five times the number:	$5x$	

$3x+4=5x+8.$	Subtract $3x$ from *both* sides.
$3x+4-3x=5x+8-3x$	
$4=2\text{x}+8$	Subtract 8 from *both* sides.
$4-8=2\text{x}+8-8$	
$-4=2\text{x}$	Divide *both* sides by 2.
$-2=x$	

Check this result. Three times -2 is -6. Increasing -6 by 4 results in $-6+4=-2$. Now, five times -2 is -10. Increasing -10 by 8 results in $-10+8=-2$. The results agree and the solution checks.

The number is -2

Example:

Consecutive integers have the property that if

$$\begin{aligned} n &= \text{the smallest integer, then} \\ n+1 &= \text{the next integer, and} \\ n+2 &= \text{the next integer, and so on.} \end{aligned}$$

Consecutive odd or even integers have the property that if

$$\begin{aligned} n &= \text{the smallest integer, then} \\ n+2 &= \text{the next odd or even integer (since odd or even numbers differ by 2), and} \\ n+4 &= \text{the next odd or even integer, and so on.} \end{aligned}$$

The sum of three consecutive odd integers is equal to one less than twice the first odd integer. Find the three integers.

$$\begin{aligned} \text{Let} \qquad n &= \text{the first odd integer. Then,} \\ n+2 &= \text{the second odd integer, and} \\ n+4 &= \text{the third odd integer.} \end{aligned}$$

Translate the words to mathematical symbols and construct an equation. Read phrases.	The sum of:	add some numbers	
	three consecutive odd integers:	n,$n+2$,$n+4$	
	is equal to:	$=$	$n+(n+2)+(n+4)=2n-1$
	one less than:	subtract 1 from	
	twice the first odd integer:	$2n$	

$n+n+2+n+4=2n-1$	
$3n+6=2n-1$	Subtract $2n$ from *both* sides.
$3n+6-2n=2n-1-2n$	
$n+6=-1$	Subtract 6 from *both* sides.
$n+6-6=-1-6$	
$n=-7$	The first integer is -7.
$n+2=-7+2=-5$	The second integer is -5.
$n+4=-7+4=-3$	The third integer is -3.

Check this result. The sum of the three integers is

$$\begin{aligned} -7+(-5)+(-3) &= -12+(-3) \\ &= -15 \end{aligned}$$

One less than twice the first integer is $2(-7)-1=-14-1=-15$. Since these two results are equal, the solution checks.

The three odd integers are -7, -5, -3.

Practice Set A

Exercise:

Problem: When three times a number is decreased by 5, the result is -23. Find the number.

Let $x =$
Check:
The number is.

Solution:

-6

Exercise:

Problem:

When five times a number is increased by 7, the result is five less than seven times the number. Find the number.

Let$n =$
Check:
The number is.

Solution:

6

Exercise:

Problem: Two consecutive numbers add to 35. Find the numbers.

Check:
The numbers areand.

Solution:

17 and 18

Exercise:

Problem:

The sum of three consecutive even integers is six more than four times the middle integer. Find the integers.

Let$x =$smallest integer.= next integer.= largest integer.
Check:
The integers are,, and.

Solution:

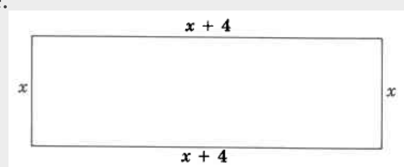
-8, -6, and -4

Geometry Problems

Sample Set B

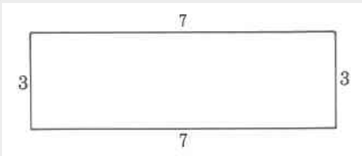
Example:
The perimeter (length around) of a rectangle is 20 meters. If the length is 4 meters longer than the width, find the length and width of the rectangle.

Let$x =$the width of the rectangle. Then,$x + 4 =$the length of the rectangle.
We can draw a picture.



The length around the rectangle is
$$\underbrace{x}_{\text{width}} + \underbrace{(x+4)}_{\text{length}} + \underbrace{x}_{\text{width}} + \underbrace{(x+4)}_{\text{length}} = 20$$

$x + x + 4 + x + x + 4 = 20$
$4x + 8 = 20$ Subtract 8 from *both* sides.
$4x = 12$ Divide *both* sides by 4.
$x = 3$ Then,
$x + 4 = 3 + 4 = 7$
Check:



$3 + 7 + 3 + 7 \stackrel{?}{=} 20$
$20 \stackrel{\checkmark}{=} 20$

The length of the rectangle is 7 meters.The width of the rectangle is 3 meters.

Practice Set B

Exercise:

Problem:

The perimeter of a triangle is 16 inches. The second leg is 2 inches longer than the first leg, and the third leg is 5 inches longer than the first leg. Find the length of each leg.

Let $x =$length of the first leg.= length of the second leg.= length of the third leg.
We can draw a picture.
Check:
The lengths of the legs are,, and.

Solution:

3 inches, 5 inches, and 8 inches

Exercises

For the following 17 problems, find each solution using the five-step method.
Exercise:

Problem: What number decreased by nine is fifteen?

Let $n =$the number.
Check:
The number is.

Solution:

24
Exercise:

Problem: What number increased by twelve is twenty?

$n =$

Let the number.
Check:
The number is.

Exercise:

Problem: If five more than three times a number is thirty-two, what is the number?

Let $x =$ the number.
Check:
The number is.

Solution:

9

Exercise:

Problem: If four times a number is increased by fifteen, the result is five. What is the number?

Let $x =$
Check:
The number is.

Exercise:

Problem:

When three times a quantity is decreased by five times the quantity, the result is negative twenty. What is the quantity?

Let $x =$
Check:
The quantity is.

Solution:

10

Exercise:

Problem:

If four times a quantity is decreased by nine times the quantity, the result is ten. What is the quantity?

Let $y =$
Check:
The quantity is.

Exercise:

Problem:

When five is added to three times some number, the result is equal to five times the number decreased by seven. What is the number?

Let $n =$

Check:
The number is.

Solution:

6

Exercise:

Problem:

When six times a quantity is decreased by two, the result is six more than seven times the quantity. What is the quantity?

Let$x =$
Check:
The quantity is.

Exercise:

Problem:

When four is decreased by three times some number, the result is equal to one less than twice the number. What is the number?

Check:

Solution:

1

Exercise:

Problem:

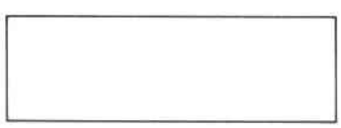
When twice a number is subtracted from one, the result is equal to twenty-one more than the number. What is the number?

Exercise:

Problem:

The perimeter of a rectangle is 36 inches. If the length of the rectangle is 6 inches more than the width, find the length and width of the rectangle.

Let$w =$the width.= the length.
We can draw a picture.



Check:
The length of the rectangle isinches, and the width isinches.

Solution:

Length=12 inches, Width=6 inches

Exercise:

Problem:

The perimeter of a rectangle is 48 feet. Find the length and the width of the rectangle if the length is 8 feet more than the width.

Let $w =$the width.= the length.
We can draw a picture.

Check:
The length of the rectangle isfeet, and the width isfeet.

Exercise:

Problem: The sum of three consecutive integers is 48. What are they?

Let $n =$the smallest integer.= the next integer.= the next integer.
Check:
The three integers are,, and.

Solution:

15, 16, 17

Exercise:

Problem: The sum of three consecutive integers is -27. What are they?

Let $n =$the smallest integer.= the next integer.= the next integer.
Check:
The three integers are,, and.

Exercise:

Problem: The sum of five consecutive integers is zero. What are they?

Let $n =$
The five integers are,,,, and.

Solution:

-2, -1, 0, 1, 2

Exercise:

Problem: The sum of five consecutive integers is -5. What are they?

Let $n =$

The five integers are,,,, and.

Continue using the five-step procedure to find the solutions.

Exercise:

Problem:

The perimeter of a rectangle is 18 meters. Find the length and width of the rectangle if the length is 1 meter more than three times the width.

Solution:

Length is 7, width is 2

Exercise:

Problem:

The perimeter of a rectangle is 80 centimeters. Find the length and width of the rectangle if the length is 2 meters less than five times the width.

Exercise:

Problem:

Find the length and width of a rectangle with perimeter 74 inches, if the width of the rectangle is 8 inches less than twice the length.

Solution:

Length is 15, width is 22

Exercise:

Problem:

Find the length and width of a rectangle with perimeter 18 feet, if the width of the rectangle is 7 feet less than three times the length.

Exercise:

Problem:

A person makes a mistake when copying information regarding a particular rectangle. The copied information is as follows: The length of a rectangle is 5 inches less than two times the width. The perimeter of the rectangle is 2 inches. What is the mistake?

Solution:

The perimeter is 20 inches. Other answers are possible. For example, perimeters such as 26, 32 are possible.

Exercise:

Problem:

A person makes a mistake when copying information regarding a particular triangle. The copied information is as follows: Two sides of a triangle are the same length. The third side is 10 feet less than three times the length of one of the other sides. The perimeter of the triangle is 5 feet. What is the mistake?

Exercise:

Problem:

The perimeter of a triangle is 75 meters. If each of two legs is exactly twice the length of the shortest leg, how long is the shortest leg?

Solution:

15 meters

Exercise:

Problem:

If five is subtracted from four times some number the result is negative twenty-nine. What is the number?

Exercise:

Problem: If two is subtracted from ten times some number, the result is negative two. What is the number?

Solution:

$n = 0$

Exercise:

Problem:

If three less than six times a number is equal to five times the number minus three, what is the number?

Exercise:

Problem:

If one is added to negative four times a number the result is equal to eight less than five times the number. What is the number?

Solution:

$n = 1$

Exercise:

Problem: Find three consecutive integers that add to -57.

Exercise:

Problem: Find four consecutive integers that add to negative two.

Solution:

-2, -1, 0, 1

Exercise:

Problem: Find three consecutive even integers that add to -24.

Exercise:

Problem: Find three consecutive odd integers that add to -99.

Solution:

-35, -33, -31

Exercise:

Problem:

Suppose someone wants to find three consecutive odd integers that add to 120. Why will that person not be able to do it?

Exercise:

Problem:

Suppose someone wants to find two consecutive even integers that add to 139. Why will that person not be able to do it?

Solution:

…because the sum of any even number (in this case, 2) o even integers (consecutive or not) is even and, therefore, cannot be odd (in this case, 139)

Exercise:

Problem:

Three numbers add to 35. The second number is five less than twice the smallest. The third number is exactly twice the smallest. Find the numbers.

Exercise:

Problem:

Three numbers add to 37. The second number is one less than eight times the smallest. The third number is two less than eleven times the smallest. Find the numbers.

Solution:

2, 15, 20

Exercises for Review

Exercise:

Problem: ([link]) Find the decimal representation of $0.34992 \div 4.32$.

Exercise:

Problem:

([link]) A 5-foot woman casts a 9-foot shadow at a particular time of the day. How tall is a person that casts a 10.8-foot shadow at the same time of the day?

Solution:

6 feet tall

Exercise:

Problem: ([link]) Use the method of rounding to estimate the sum: $4\frac{5}{12} + 15\frac{1}{25}$.

Exercise:

Problem: ([link]) Convert 463 mg to cg.

Solution:

46.3 cg

Exercise:

Problem:

([link]) Twice a number is added to 5. The result is 2 less than three times the number. What is the number?

Simplifying Algebraic Expressions Using Addition and Subtraction
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to combine like terms using addition and subtraction. By the end of the module students should be able to combine like terms in an algebraic expression.

Section Overview

- Combining Like Terms

Combining Like Terms

From our examination of terms in [link], we know that **like terms** are terms in which the variable parts are identical. Like terms is an appropriate name since terms with identical variable parts and different numerical coefficients represent different amounts of the same quantity. When we are dealing with quantities of the same type, we may combine them using addition and subtraction.

Simplifying an Algebraic Expression
An algebraic expression may be **simplified** by combining like terms.

This concept is illustrated in the following examples.

1. $8 \text{ records} + 5 \text{ records} = 13 \text{ records}.$

   Eight and 5 of the same type give 13 of that type. We have combined quantities of the same type.
2. $8 \text{ records} + 5 \text{ records} + 3 \text{ tapes} = 13 \text{ records} + 3 \text{ tapes}.$

   Eight and 5 of the same type give 13 of that type. Thus, we have 13 of one type and 3 of another type. We have combined only quantities of the same type.
3. Suppose we let the letter x represent "record." Then, $8x + 5x = 13x$. The terms $8x$ and $5x$ are like terms. So, 8 and 5 of the same type give 13 of that type. We have combined like terms.

4. Suppose we let the letter x represent "record" and y represent "tape." Then,

   $$8x + 5x + 3y = 13x + 5y$$

   We have combined only the like terms.

After observing the problems in these examples, we can suggest a method for simplifying an algebraic expression by combining like terms.

Combining Like Terms
Like terms may be combined by adding or subtracting their coefficients and affixing the result to the common variable.

Sample Set A

Simplify each expression by combining like terms.

Example:
$2m + 6m - 4m$. All three terms are alike. Combine their coefficients and affix this result to m: $2 + 6 - 4 = 4$.
Thus, $2m + 6m - 4m = 4m$.

Example:
$5x + 2y - 9y$. The terms $2y$ and $-9y$ are like terms. Combine their coefficients: $2 - 9 = -7$.
Thus, $5x + 2y - 9y = 5x - 7y$.

Example:
$-3a + 2b - 5a + a + 6b$. The like terms are

$-3a, \ -5a, a \ 2b, 6b$

$\underbrace{\underbrace{}_{-3-5+1=-7}}_{-7a}$ $\underbrace{\underbrace{}_{2+6=8}}_{8b}$

Thus, $-3a+2b-5a+a+6b{=}-7a+8b$.

Example:

$r-2s+7s+3r-4r-5s$. The like terms are

$$\underbrace{\underbrace{\underbrace{r, 3r, -4r}_{1+3-4=0}}_{0r} \qquad \underbrace{\underbrace{-2s, 7s, -5s}_{-2+7-5=0}}_{0s}}_{0r+0s=0}$$

Thus, $r-2s+7s+3r-4r-5s=0$.

Practice Set A

Simplify each expression by combining like terms.

Exercise:

Problem: $4x+3x+6x$

Solution:

$13x$

Exercise:

Problem: $5a+8b+6a-2b$

Solution:

$11a+6b$

Exercise:

Problem: $10m - 6n - 2n - m + n$

Solution:

$9m - 7n$

Exercise:

Problem: $16a + 6m + 2r - 3r - 18a + m - 7m$

Solution:

$-2a - r$

Exercise:

Problem: $5h - 8k + 2h - 7h + 3k + 5k$

Solution:

0

Exercises

Simplify each expression by combining like terms.

Exercise:

Problem: $4a + 7a$

Solution:

$11a$

Exercise:

Problem: $3m + 5m$

Exercise:

Problem: $6h - 2h$

Solution:

$4h$

Exercise:

Problem: $11k - 8k$

Exercise:

Problem: $5m + 3n - 2m$

Solution:

$3m + 3n$

Exercise:

Problem: $7x - 6x + 3y$

Exercise:

Problem: $14s + 3s - 8r + 7r$

Solution:

$17s - r$

Exercise:

Problem: $-5m - 3n + 2m + 6n$

Exercise:

Problem: $7h + 3a - 10k + 6a - 2h - 5k - 3k$

Solution:

$5h+9a-18k$

Exercise:

Problem: $4x-8y-3z+x-y-z-3y-2z$

Exercise:

Problem: $11w+3x-6w-5w+8x-11x$

Solution:

0

Exercise:

Problem: $15r-6s+2r+8s-6r-7s-s-2r$

Exercise:

Problem: $\mid -7 \mid m+ \mid 6 \mid m+ \mid -3 \mid m$

Solution:

$16m$

Exercise:

Problem: $\mid -2 \mid x+ \mid -8 \mid x+ \mid 10 \mid x$

Exercise:

Problem: $(-4+1)k+(6-3)k+(12-4)h+(5+2)k$

Solution:

$8h + 7k$

Exercise:

Problem: $(-5+3)a - (2+5)b - (3+8)b$

Exercise:

Problem: $5\star + 2\mathit{\Delta} + 3\mathit{\Delta} - 8\star$

Solution:

$5\mathit{\Delta} - 3\star$

Exercise:

Problem: $9 \quad + 10 \quad - 11 \quad - 12$

Exercise:

Problem: $16x - 12y + 5x + 7 - 5x - 16 - 3y$

Solution:

$16x - 15y - 9$

Exercise:

Problem: $-3y + 4z - 11 - 3z - 2y + 5 - 4(8-3)$

Exercises for Review

Exercise:

Problem: ([link]) Convert $\frac{24}{11}$ to a mixed number

Solution:

$2\frac{2}{11}$

Exercise:

Problem: ([link]) Determine the missing numerator: $\frac{3}{8} = \frac{}{64}$.

Exercise:

Problem: ([link]) Simplify $\frac{\frac{5}{6} - \frac{1}{4}}{\frac{1}{12}}$.

Solution:

7

Exercise:

Problem: ([link]) Convert $\frac{5}{16}$ to a percent.

Exercise:

Problem: ([link]) In the expression $6k$, how many k's are there?

Solution:

6

Solving Linear Equations: The Addition Property
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to solve equations of the form $x + a = b$ and $x - a = b$. By the end of the module students should understand the meaning and function of an equation, understand what is meant by the solution to an equation and be able to solve equations of the form $x + a = b$ and $x - a = b$.

Section Overview

- Equations
- Solutions and Equivalent Equations
- Solving Equations

Equations

Equation

An equation is a statement that two algebraic expressions are equal.

The following are examples of equations:

$$\begin{array}{ccccccccc} x+6 & = & 10 & x-4 & = & -11 & 3y-5 & = & -2+2y \\ \text{This expression} & = & \text{This expression} & \text{This expression} & = & \text{This expression} & \text{This expression} & = & \text{This expression} \end{array}$$

Notice that $x + 6$, $x - 4$, and $3y - 5$ are *not* equations. They are expressions. They are not equations because there is no statement that each of these expressions is equal to another expression.

Solutions and Equivalent Equations

Conditional Equations

The truth of some equations is conditional upon the value chosen for the variable. Such equations are called **conditional equations**. There are two additional types of equations. They are examined in courses in algebra, so we will not consider them now.

Solutions and Solving an Equation

The set of values that, when substituted for the variables, make the equation true, are called the **solutions** of the equation.
An equation has been **solved** when all its solutions have been found.

Sample Set A

Example:
Verify that 3 is a solution to $x+7=10$.
When $x=3$,

$$\begin{array}{rrcll} & x+7 & = & 10 & \\ \text{becomes} & 3+7 & = & 10 & \\ & 10 & = & 10 & \text{which is a } \textit{true} \text{ statement, verifying that} \\ & & & & 3 \text{ is a solution to } x+7=10 \end{array}$$

Example:
Verify that -6 is a solution to $5y+8=-22$
When $y=-6$,

$$\begin{array}{rrcll} & 5y+8 & = & -22 & \\ \text{becomes} & 5(-6)+8 & = & -22 & \\ & -30+8 & = & -22 & \\ & -22 & = & -22 & \text{which is a } \textit{true} \text{ statement, verifying that} \\ & & & & -6 \text{ is a solution to} 5y+8=-22 \end{array}$$

Example:
Verify that 5 is not a solution to $a-1=2a+3$.
When $a=5$,

$$\begin{array}{rrcll} & a-1 & = & 2a+3 & \\ \text{becomes} & 5-1 & = & 2\cdot 5+3 & \\ & 5-1 & = & 10+3 & \\ & 4 & = & 13 & \text{a } \textit{false} \text{ statement, verifying that } 5 \\ & & & & \text{is not a solution to } a-1=2a+3 \end{array}$$

Example:
Verify that -2 is a solution to $3m-2=-4m-16$.
When $m=-2$,

$$\begin{array}{rrcll} & 3m-2 & = & -4m-16 & \\ \text{becomes} & 3(-2)-2 & = & -4(-2)-16 & \\ & -6-2 & = & 8-16 & \\ & -8 & = & -8 & \text{which is a } \textit{true} \text{ statement, verifying that} \\ & & & & -2 \text{ is a solution to } 3m-2=-4m-16 \end{array}$$

Practice Set A

Exercise:

Problem: Verify that 5 is a solution to $m + 6 = 11$.

Solution:

Substitute 5 into $m + 6 = 11$.

$$\begin{aligned} 5 + 6 &\stackrel{?}{=} 11 \\ 11 &\stackrel{\checkmark}{=} 11 \end{aligned}$$

Thus, 5 is a solution.

Exercise:

Problem: Verify that -5 is a solution to $2m - 4 = -14$.

Solution:

Substitute -5 into $2m - 4 = -14$.

$$\begin{aligned} 2(-5) - 4 &\stackrel{?}{=} -14 \\ -10 - 4 &\stackrel{?}{=} -14 \\ -14 &\stackrel{\checkmark}{=} -14 \end{aligned}$$

Thus, -5 is a solution.

Exercise:

Problem: Verify that 0 is a solution to $5x + 1 = 1$.

Solution:

Substitute 0 into $5x + 1 = 1$.

$$\begin{aligned} 5(0) + 1 &\stackrel{?}{=} 1 \\ 0 + 1 &\stackrel{?}{=} 1 \\ 1 &\stackrel{\checkmark}{=} 1 \end{aligned}$$

Thus, 0 is a solution.

Exercise:

Problem: Verify that 3 is not a solution to $-3y + 1 = 4y + 5$.

Solution:

Substitute 3 into $-3y + 1 = 4y + 5$.

$$\begin{aligned} -3(3) + 1 &\stackrel{?}{=} 4(3) + 5 \\ -9 + 1 &\stackrel{?}{=} 12 + 5 \\ -8 &\neq 17 \end{aligned}$$

Thus, 3 is not a solution.

Exercise:

Problem: Verify that -1 is a solution to $6m - 5 + 2m = 7m - 6$.

Solution:

Substitute -1 into $6m - 5 + 2m = 7m - 6$.

$$\begin{aligned} 6(-1) - 5 + 2(-1) &\stackrel{?}{=} 7(-1) - 6 \\ -6 - 5 - 2 &\stackrel{?}{=} -7 - 6 \\ -13 &\stackrel{\checkmark}{=} -13 \end{aligned}$$

Thus, -1 is a solution.

Equivalent Equations

Some equations have precisely the same collection of solutions. Such equations are called equivalent equations. For example, $x - 5 = -1$, $x + 7 = 11$, and $x = 4$ are all equivalent equations since the only solution to each is $x = 4$. (Can you verify this?)

Solving Equations

We know that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side.

This number	is the same as	this number
↓	↓	↓
x	=	4

$x + 7$	=	11
$x - 5$	=	-1

Addition/Subtraction Property of Equality
From this, we can suggest the **addition/subtraction property of equality**.
Given any equation,

1. We can obtain an equivalent equation by *adding* the *same* number to *both* sides of the equation.
2. We can obtain an equivalent equation by *subtracting* the *same* number from *both* sides of the equation.

The Idea Behind Equation Solving
The idea behind **equation solving** is to isolate the variable on one side of the equation. Signs of operation (+, -, ·,÷) are used to associate two numbers. For example, in the expression $5 + 3$, the numbers 5 and 3 are associated by addition. An association can be *undone* by performing the opposite operation. The addition/subtraction property of equality can be used to undo an association that is made by addition or subtraction.

Subtraction is used to undo an addition.

Addition is used to undo a subtraction.

The procedure is illustrated in the problems of [link].

Sample Set B

Use the addition/subtraction property of equality to solve each equation.

Example:
$x + 4 = 6.$
4 is associated with x by addition. Undo the association by *subtracting* 4 from *both* sides.
$x + 4 - 4 = 6 - 4$

$x + 0 = 2$

$x = 2$
Check: When $x = 2$, $x + 4$ becomes

$$2 + 4 \stackrel{?}{=} 6$$
$$6 \stackrel{\checkmark}{=} 6.$$

The solution to $x + 4 = 6$ is $x = 2$.

Example:
$m - 8 = 5$. 8 is associated with m by subtraction. Undo the association by *adding* 8 to *both* sides.
$m - 8 + 8 = 5 + 8$

$m + 0 = 13$

$m = 13$
Check: When $m = 13$,
becomes

$$\begin{aligned} m - 8 &= 5 \\ 13 - 8 &\overset{?}{=} 5 \\ 5 &\overset{\checkmark}{=} 5 \end{aligned}$$

a true statement.
The solution to $m - 8 = 5$ is $m = 13$.

Example:
$-3 - 5 = y - 2 + 8$. Before we use the addition/subtraction property, we should simplify as much as possible.
$-3 - 5 = y - 2 + 8$
$-8 = y + 6$
6 is associated with y by addition. Undo the association by *subtracting* 6 from *both* sides.
$-8 - 6 = y + 6 - 6$

$-14 = y + 0$

$-14 = y$
This is equivalent to $y = -14$.
Check: When $y = -14$,
$-3 - 5 = y - 2 + 8$
becomes

$$\begin{aligned} -3 - 5 &\overset{?}{=} -14 - 2 + 8 \\ -8 &\overset{?}{=} -16 + 8 \\ -8 &\overset{\checkmark}{=} -8. \end{aligned}$$

,
a true statement.
The solution to $-3 - 5 = y - 2 + 8$ is $y = -14$.

Example:
$-5\text{a}+1+6\text{a}=-2$. Begin by simplifying the left side of the equation.
$-5a+1+6a=-2$

$-5+6=1$

$a+1=-2$ 1 is associated with a by addition. Undo the association by *subtracting* 1 from *both* sides.
$a+1-1=-2-1$

$a+0=-3$

$a=-3$
Check: When $a=-3$,
$-5\text{a}+1+6\text{a}=-2$
becomes

$$\begin{aligned} -5(-3)+1+6(-3) &\stackrel{?}{=} -2 \\ 15+1-18 &\stackrel{?}{=} -2 \\ -2 &\stackrel{\checkmark}{=} -2 \end{aligned}$$

,
a true statement.
The solution to $-5a+1+6a=-2$ is $a=-3$.

Example:
$7k-4=6k+1$. In this equation, the variable appears on both sides. We need to isolate it on one side. Although we can choose either side, it will be more convenient to choose the side with the larger coefficient. Since 8 is greater than 6, we'll isolate k on the left side.
$7k-4=6k+1$ Since $6k$ represents $+6k$, subtract $6k$ from each side.
$7k-4-6k=6k+1-6k$

$7-6=1$ $6-6=0$

$k-4=1$ 4 is associated with k by subtraction. Undo the association by *adding* 4 to *both* sides.
$k-4+4=1+4$

$k=5$
Check: When $k=5$,
$7k-4=6k+1$
becomes

$$\begin{aligned} 7\cdot 5-4 &\stackrel{?}{=} 6\cdot 5+1 \\ 35-4 &\stackrel{?}{=} 30+1 \\ 31 &\stackrel{\checkmark}{=} 31. \end{aligned}$$

a true statement.
The solution to $7k-4=6k+1$ is $k=5$.

Example:
$-8+x=5$. -8 is associated with x by addition. Undo the by *subtracting* -8 from *both* sides. Subtracting -8 we get $-(-8)$=+8. We actually *add* 8 to both sides.
$-8+x+8=5+8$
$x=13$
Check: When $x=13$
$-8+x=5$
becomes

$$\begin{aligned} -8+13 &\stackrel{?}{=} 5 \\ 5 &\stackrel{\checkmark}{=} 5 \end{aligned}$$

,
a true statement.
The solution to $-8+x=5$ is $x=13$.

Practice Set B

Exercise:

Problem: $y+9=4$

Solution:

$y=-5$

Exercise:

Problem: $a-4=11$

Solution:

$a=15$

Exercise:

Problem: $-1+7=x+3$

Solution:

$x = 3$

Exercise:

Problem: $8m + 4 - 7m = (-2)(-3)$

Solution:

$m = 2$

Exercise:

Problem: $12k - 4 = 9k - 6 + 2k$

Solution:

$k = -2$

Exercise:

Problem: $-3 + a = -4$

Solution:

$a = -1$

Exercises

For the following 10 problems, verify that each given value is a solution to the given equation.

Exercise:

Problem: $x - 11 = 5$, $x = 16$

Solution:

Substitute $x = 4$ into the equation $4x - 11 = 5$.
$16 - 11 = 5$

$5 = 5$
$x = 4$ is a solution.

Exercise:

Problem: $y - 4 = -6, y = -2$

Exercise:

Problem: $2m - 1 = 1, m = 1$

Solution:

Substitute $m = 1$ into the equation $2m - 1 = 1$.

$$\begin{aligned} 2 - 1 &\stackrel{?}{=} 1 \\ 1 &\stackrel{\checkmark}{=} 1 \end{aligned}$$

$m = 1$ is a solution.

Exercise:

Problem: $5y + 6 = -14, y = -4$

Exercise:

Problem: $3\text{x} + 2 - 7\text{x} = -5\text{x} - 6, x = -8$

Solution:

Substitute $x = -8$ into the equation $3\text{x} + 2 - 7 = -5\text{x} - 6$.

$$-24 + 2 - 7 \stackrel{?}{=} 40 - 6$$
$$34 \stackrel{\checkmark}{=} 34$$

$x = -8$ is a solution.

Exercise:

Problem: $-6a + 3 + 3a = 4a + 7 - 3a, a = -1$

Exercise:

Problem: $-8 + x = -8, x = 0$

Solution:

Substitute $x = 0$ into the equation $-8 + x = -8$.

$$-8 + 0 \stackrel{?}{=} -8$$
$$-8 \stackrel{\checkmark}{=} -8$$

$x = 0$ is a solution.

Exercise:

Problem: $8b + 6 = 6 - 5b$, $b = 0$

Exercise:

Problem: $4x - 5 = 6x - 20$, $x = \frac{15}{2}$

Solution:

Substitute $x = \frac{15}{2}$ into the equation $4x - 5 = 6x - 20$.

$$30 - 5 \stackrel{?}{=} 45 - 20$$
$$25 \stackrel{\checkmark}{=} 25$$

$x = \frac{15}{2}$ is a solution.

Exercise:

Problem: $-3y + 7 = 2y - 15$, $y = \frac{22}{5}$

Solve each equation. Be sure to check each result.

Exercise:

Problem: $y - 6 = 5$

Solution:

$y = 11$

Exercise:

Problem: $m + 8 = 4$

Exercise:

Problem: $k - 1 = 4$

Solution:

$k = 5$

Exercise:

Problem: $h - 9 = 1$

Exercise:

Problem: $a + 5 = -4$

Solution:

$a = -9$

Exercise:

Problem: $b - 7 = -1$

Exercise:

Problem: $x + 4 - 9 = 6$

Solution:

$x = 11$

Exercise:

Problem: $y - 8 + 10 = 2$

Exercise:

Problem: $z + 6 = 6$

Solution:

$z = 0$

Exercise:

Problem: $w - 4 = -4$

Exercise:

Problem: $x + 7 - 9 = 6$

Solution:

$x = 8$

Exercise:

Problem: $y - 2 + 5 = 4$

Exercise:

Problem: $m + 3 - 8 = -6 + 2$

Solution:

$m = 1$

Exercise:

Problem: $z + 10 - 8 = -8 + 10$

Exercise:

Problem: $2 + 9 = k - 8$

Solution:

$k = 19$

Exercise:

Problem: $-5 + 3 = h - 4$

Exercise:

Problem: $3m - 4 = 2m + 6$

Solution:

$m = 10$

Exercise:

Problem: $5a + 6 = 4a - 8$

Exercise:

Problem: $8b + 6 + 2b = 3b - 7 + 6b - 8$

Solution:

$b = -21$

Exercise:

Problem: $12h - 1 - 3 - 5h = 2h + 5h + 3(-4)$

Exercise:

Problem: $-4a + 5 - 2a = -3a - 11 - 2a$

Solution:

$a = 16$

Exercise:

Problem: $-9n - 2 - 6 + 5n = 3n - (2)(-5) - 6n$

Calculator Exercises

Exercise:

Problem: $y - 2.161 = 5.063$

Solution:

$y = 7.224$

Exercise:

Problem: $a - 44.0014 = -21.1625$

Exercise:

Problem: $-0.362 - 0.416 = 5.63m - 4.63m$

Solution:

$m = -0.778$

Exercise:

Problem: $8.078 - 9.112 = 2.106y - 1.106y$

Exercise:

Problem: $4.23k + 3.18 = 3.23k - 5.83$

Solution:

$k = -9.01$

Exercise:

Problem: $6.1185x - 4.0031 = 5.1185x - 0.0058$

Exercise:

Problem: $21.63y + 12.40 - 5.09y = 6.11y - 15.66 + 9.43y$

Solution:

$y = -28.06$

Exercise:

Problem: $0.029a - 0.013 - 0.034 - 0.057 = -0.038 + 0.56 + 1.01a$

Exercises for Review

Exercise:

Problem: ([link]) Is $\frac{7 \text{calculators}}{12 \text{students}}$ an example of a ratio or a rate?

Solution:

rate

Exercise:

Problem: ([link]) Convert $\frac{3}{8}\%$ to a decimal.

Exercise:

Problem: ([link]) 0.4% of what number is 0.014?

Solution:

3.5

Exercise:

Problem:

([link]) Use the clustering method to estimate the sum: $89 + 93 + 206 + 198 + 91$

Exercise:

Problem: ([link]) Combine like terms: $4x + 8y + 12y + 9x - 2y$.

Solution:

$13x + 18y$

Solving Linear Equations: The Multiplication Property
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter, the emphasis is on the mechanics of equation solving, which clearly explains how to isolate a variable. The goal is to help the student feel more comfortable with solving applied problems. Ample opportunity is provided for the student to practice translating words to symbols, which is an important part of the "Five-Step Method" of solving applied problems (discussed in modules (<link document="m21980"/>) and (<link document="m21979"/>)). Objectives of this module: understand the equality property of addition and multiplication, be able to solve equations of the form ax = b and x/a = b.

Overview

- Equality Property of Division and Multiplication
- Solving $ax = b$ and $\frac{x}{a} = b$ for x

Equality Property of Division and Multiplication

Recalling that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side suggests the equality property of division and multiplication, which states:

1. We can obtain an equivalent equation by **dividing both sides** of the equation by the same nonzero number, that is, if $c \neq 0$, then $a = b$ is equivalent to $\frac{a}{c} = \frac{b}{c}$.
2. We can obtain an equivalent equation by **multiplying both sides** of the equation by the same nonzero number, that is, if $c \neq 0$, then $a = b$ is equivalent to $ac = bc$.

We can use these results to isolate x, thus solving the equation for x.

Example:
Solving $ax = b$ for x

$$\begin{array}{rcll} ax & = & b & a \text{ is associated with } x \text{ by multiplication.} \\ & & & \text{Undo the association by dividing both sides by} a. \\ \frac{ax}{a} & = & \frac{b}{a} & \\ \frac{\cancel{a}x}{a} & = & \frac{b}{a} & \\ 1 \cdot x & = & \frac{b}{a} & \frac{a}{a} = 1 \text{ and } 1 \text{ is the multiplicative identity. } 1 \cdot x = x \end{array}$$

Example:
Solving $\frac{x}{a} = b$ for x

$$\begin{array}{rcll} x & = & \frac{b}{a} & \text{This equation is equivalent to the first and is solved by} x. \\ \frac{x}{a} & = & b & a \text{ is associated with } x \text{ by division. Undo the association by multiplying both sides by} a. \\ a \cdot \frac{x}{a} & = & a \cdot b & \\ \cancel{a} \cdot \frac{x}{\cancel{a}} & = & ab & \\ 1 \cdot x & = & ab & \frac{a}{a} = 1 \text{ and } 1 \text{ is the multiplicative identity. } 1 \cdot x = x \\ x & = & ab & \text{This equation is equivalent to the first and is solved for } x. \end{array}$$

Solving $ax = b$ and $\frac{x}{a} = b$ for x

Example:
Method for Solving $ax = b$ and $\frac{x}{a} = b$
To solve $ax = b$ for x, **divide both sides** of the equation by a.
To solve $\frac{x}{a} = b$ for x, **multiply both sides** of the equation by a.

Sample Set A

Example:
Solve $5x = 35$ for x.

$$\begin{array}{rcll} 5x & = & 35 & 5 \text{ is associated with x by multiplication. Undo the association by dividing both sides by } 5. \\ \frac{5x}{5} & = & \frac{35}{5} & \\ \frac{\cancel{5}x}{\cancel{5}} & = & 7 & \\ 1 \cdot x & = & 7 & \frac{5}{5} = 1 \text{ and } 1 \text{ is multiplicative identity. } 1 \cdot x = x. \\ x & = & 7 & \end{array}$$

$$\begin{array}{lrcll} Check: & 5(7) & = & 35 & \text{Is this correct?} \\ & 35 & = & 35 & \text{Yes, this is correct.} \end{array}$$

Example:

Solve $\frac{x}{4} = 5$ for x.

$$\frac{x}{4} = 5$$ 4 is asssociated with x by division. Undo the association by multiplying *both* sides by 4.

$$4 \cdot \frac{x}{4} = 4 \cdot 5$$

$$\cancel{4} \cdot \frac{x}{\cancel{4}} = 4 \cdot 5$$

$$1 \cdot x = 20$$ $\frac{4}{4} = 1$ and 1 is the multiplicative identity. $1 \cdot x = x$.

$$x = 20$$

$Check:$ $\frac{20}{4} = 5$ Is this correct?

$5 = 5$ Yes, this is correct.

Example:

Solve $\frac{2y}{9} = 3$ for y.

Method (1) (Use of cancelling):

$$\frac{2y}{9} = 3$$ 9 is associated with y by division. Undo the association by multiplying *both* sides by 9.

$$(\cancel{9})\left(\frac{2y}{\cancel{9}}\right) = (9)(3)$$

$$2y = 27$$ 2 is associated with y by multiplication. Undo the association by dividing *both* sides by 2.

$$\frac{\cancel{2}y}{\cancel{2}} = \frac{27}{2}$$

$$y = \frac{27}{2}$$

$Check:$ $\frac{\cancel{2}\left(\frac{27}{\cancel{2}}\right)}{9} = 3$ Is this correct?

$\frac{27}{9} = 3$ Is this correct?

$3 = 3$ Yes, this is correct.

Method (2) (Use of reciprocals):

$$\frac{2y}{9} = 3$$ Since $\frac{2y}{9} = \frac{2}{9}y$, $\frac{2}{9}$ is associated with y by multiplication. Then, Since $\frac{9}{2} \cdot \frac{2}{9} = 1$, the multiplicative identity, we can undo the associative by multiplying *both* sides by $\frac{9}{2}$.

$$\left(\frac{9}{2}\right)\left(\frac{2y}{9}\right) = \left(\frac{9}{2}\right)(3)$$

$$\left(\frac{9}{2} \cdot \frac{2}{9}\right)y = \frac{27}{2}$$

$$1 \cdot y = \frac{27}{2}$$

$$y = \frac{27}{2}$$

Example:

Solve the literal equation $\frac{4ax}{m} = 3b$ for x.

$$\begin{array}{rcll} \frac{4ax}{m} & = & 3b & m \text{ is associated with } x \text{ by division. Undo the association by} \\ & & & \text{multiplying } both \text{ sides by } m. \\ \cancel{m}\left(\frac{4ax}{\cancel{m}}\right) & = & m \cdot 3b & \\ 4ax & = & 3bm & 4a \text{ is associated with } x \text{ by multiplication. Undo the} \\ & & & \text{association by multiplying } both \text{ sides by } 4a. \\ \frac{\cancel{4a}\,x}{\cancel{4a}} & = & \frac{3bm}{4a} & \\ x & = & \frac{3bm}{4a} & \end{array}$$

$$\begin{array}{rrcll} Check: & \frac{4a\left(\frac{3bm}{4a}\right)}{m} & = & 3b & \text{Is this correct?} \\ & \frac{\cancel{4a}\left(\frac{3bm}{\cancel{4a}}\right)}{m} & = & 3b & \text{Is this correct?} \\ & \frac{3b\,\cancel{m}}{\cancel{m}} & = & 3b & \text{Is this correct?} \\ & 3b & = & 3b & \text{Yes, this is correct.} \end{array}$$

Practice Set A

Exercise:

Problem: Solve $6a = 42$ for a.

Solution:

$a = 7$

Exercise:

Problem: Solve $-12m = 16$ for m.

Solution:

$m = -\frac{4}{3}$

Exercise:

Problem: Solve $\frac{y}{8} = -2$ for y.

Solution:

$y = \ -16$

Exercise:

Problem: Solve $6.42x = 1.09$ for x.

Solution:

$x = 0.17$ (rounded to two decimal places)

Round the result to two decimal places.

Exercise:

Problem: Solve $\frac{5k}{12} = 2$ for k.

Solution:

$k = \frac{24}{5}$

Exercise:

Problem: Solve $\frac{-ab}{2c} = 4d$ for b.

Solution:

$b = \frac{-8cd}{a}$

Exercise:

Problem: Solve $\frac{3xy}{4} = 9xh$ for y.

Solution:

$y = 12h$

Exercise:

Problem: Solve $\frac{2k^2mn}{5pq} = -6n$ for m.

Solution:

$m = \frac{-15pq}{k^2}$

Exercises

In the following problems, solve each of the conditional equations.

Exercise:

Problem: $3x = 42$

Solution:

$x = 14$

Exercise:

Problem: $5y = 75$

Exercise:

Problem: $6x = 48$

Solution:

$x = 8$

Exercise:

Problem: $8x = 56$

Exercise:

Problem: $4x = 56$

Solution:

$x = 14$

Exercise:

Problem: $3x = 93$

Exercise:

Problem: $5a\ =\ -80$

Solution:

$a=-16$

Exercise:

Problem: $9m\ =\ -108$

Exercise:

Problem: $6p\ =\ -108$

Solution:

$p=-18$

Exercise:

Problem: $12q\ =\ -180$

Exercise:

Problem: $-4a\ =\ 16$

Solution:

$a=-4$

Exercise:

Problem: $-20x\ =\ 100$

Exercise:

Problem: $-6x\ =\ -42$

Solution:

$x=7$

Exercise:

Problem: $-8m\ =\ -40$

Exercise:

Problem: $-3k\ =\ 126$

Solution:

$k=-42$

Exercise:

Problem: $-9y\ =\ 126$

Exercise:

Problem: $\frac{x}{6}\ =\ 1$

Solution:

$x=6$

Exercise:

Problem: $\frac{a}{5}\ =\ 6$

Exercise:

Problem: $\frac{k}{7}\ =\ 6$

Solution:

$k=42$

Exercise:

Problem: $\frac{x}{3}\ =\ 72$

Exercise:

Problem: $\frac{x}{8}\ =\ 96$

Solution:

$x=768$

Exercise:

Problem: $\frac{y}{-3} \ = \ -4$

Exercise:

Problem: $\frac{m}{7} \ = \ -8$

Solution:

$m = -56$

Exercise:

Problem: $\frac{k}{18} \ = \ 47$

Exercise:

Problem: $\frac{f}{-62} \ = \ 103$

Solution:

$f = -6386$

Exercise:

Problem: $3.06m = \ 12.546$

Exercise:

Problem: $5.012k \ = \ 0.30072$

Solution:

$k = 0.06$

Exercise:

Problem: $\frac{x}{2.19} \ = \ 5$

Exercise:

Problem: $\frac{y}{4.11} \ = \ 2.3$

Solution:

$y = 9.453$

Exercise:

Problem: $\frac{4y}{7} \ = \ 2$

Exercise:

Problem: $\frac{3m}{10} \ = \ -1$

Solution:

$m = \frac{-10}{3}$

Exercise:

Problem: $\frac{5k}{6} \ = \ 8$

Exercise:

Problem: $\frac{8h}{-7} \ = \ -3$

Solution:

$h = \frac{21}{8}$

Exercise:

Problem: $\frac{-16z}{21} \ = \ -4$

Exercise:

Problem: Solve$pq \ = \ 7r$ for$p$.

Solution:

$p = \frac{7r}{q}$

Exercise:

Problem: Solve $m^2 n \ = \ 2s$ for n.

Exercise:

Problem: Solve $2.8ab \ = \ 5.6d$ for b.

Solution:

$b = \frac{2d}{a}$

Exercise:

Problem: Solve $\frac{mnp}{2k} = 4k$ for p.

Exercise:

Problem: Solve $\frac{-8a^2b}{3c} = -5a^2$ for b.

Solution:

$b = \frac{15c}{8}$

Exercise:

Problem: Solve $\frac{3pcb}{2m} = 2b$ for pc.

Exercise:

Problem: Solve $\frac{8rst}{3p} = -2prs$ for t.

Solution:

$t = -\frac{3p^2}{4}$

Exercise:

Solve

$$\frac{\square \cdot \star}{\triangle} = \diamond$$

Problem: for $\square$.

Exercise:

Problem: Solve $\frac{3\square\Delta\nabla}{2\nabla} = \Delta\nabla$ for $\square$.

Solution:

$\square = \frac{2\nabla}{3}$

Exercises for Review

Exercise:

Problem: ([link]) Simplify $\left(\frac{2x^0y^0z^3}{z^2}\right)^5$.

Exercise:

Problem:

([link]) Classify $10x^3 - 7x$ as a monomial, binomial, or trinomial. State its degree and write the numerical coefficient of each item.

Solution:

binomial; 3rd degree; $10, -7$

Exercise:

Problem: ([link]) Simplify $3a^2 - 2a + 4a(a+2)$.

Exercise:

Problem: ([link]) Specify the domain of the equation $y = \frac{3}{7+x}$.

Solution:

all real numbers except -7

Exercise:

Problem: ([link]) Solve the conditional equation $x + 6 = -2$.

Solving Linear Equations by Combining Properties
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter, the emphasis is on the mechanics of equation solving, which clearly explains how to isolate a variable. The goal is to help the student feel more comfortable with solving applied problems. Ample opportunity is provided for the student to practice translating words to symbols, which is an important part of the "Five-Step Method" of solving applied problems (discussed in modules (<link document="m21980"/>) and (<link document="m21979"/>)). Objectives of this module: be able to identify various types of equations, understand the meaning of solutions and equivalent equations, be able to solve equations of the form x + a = b and x - a = b, be familiar with and able to solve literal equations.

Overview

- Types of Equations
- Solutions and Equivalent Equations
- Literal Equations
- Solving Equations of the Form $x + a = b$ and $x - a = b$

Types of Equations

Identity
Some equations are always true. These equations are called identities. **Identities** are equations that are true for all acceptable values of the variable, that is, for all values in the domain of the equation.

$5x = 5x$ is true for all acceptable values of x.
$y + 1 = y + 1$ is true for all acceptable values of y.
$2 + 5 = 7$ is true, and no substitutions are necessary.

Contradiction
Some equations are never true. These equations are called contradictions. **Contradictions** are equations that are never true regardless of the value substituted for the variable.

$x = x + 1$ is never true for any acceptable value of x.
$0 \cdot k = 14$ is never true for any acceptable value of k.
$2 = 1$ is never true.

Conditional Equation
The truth of some equations is conditional upon the value chosen for the variable. Such equations are called conditional equations. **Conditional equations** are equations that are true for at least one replacement of the variable and false for at least one replacement of the variable.

$x + 6 = 11$ is true only on the condition that $x = 5$.
$y - 7 = -1$ is true only on the condition that $y = 6$.

Solutions and Equivalent Equations

Solutions and Solving an Equation

The collection of values that make an equation true are called **solutions** of the equation. An equation is **solved** when all its solutions have been found.

Equivalent Equations
Some equations have precisely the same collection of solutions. Such equations are called **equivalent equations**. The equations
$2x + 1 = 7, \quad 2x = 6 \quad \text{and} \quad x = 3$
are equivalent equations because the only value that makes each one true is 3.

Sample Set A

Tell why each equation is an identity, a contradiction, or conditional.

Example:
The equation $x - 4 = 6$ is a conditional equation since it will be true only on the condition that $x = 10$.

Example:
The equation $x - 2 = x - 2$ is an identity since it is true for all values of x. For example,

$$\begin{array}{rrcl} \text{if } x = & 5, & 5 - 2 = & 5 - 2 \text{ is true} \\ x = & -7, & -7 - 2 = & -7 - 2 \text{ is true} \end{array}$$

Example:
The equation $a + 5 = a + 1$ is a contradiction since every value of a produces a false statement. For example,

$$\begin{array}{rrcl} \text{if } a = & 8, & 8 + 5 = & 8 + 1 \text{ is false} \\ \text{if } a = & -2, & -2 + 5 = & -2 + 1 \text{ is false} \end{array}$$

Practice Set A

For each of the following equations, write "identity," "contradiction," or "conditional." If you can, find the solution by making an educated guess based on your knowledge of arithmetic.

Exercise:

Problem: $x + 1 = 10$

Solution:

conditional, $x = 9$

Exercise:

Problem: $y-4=7$

Solution:

conditional, $y=11$

Exercise:

Problem: $5a=25$

Solution:

conditional, $a=5$

Exercise:

Problem: $\frac{x}{4}=9$

Solution:

conditional, $x=36$

Exercise:

Problem: $\frac{18}{b}=6$

Solution:

conditional, $b=3$

Exercise:

Problem: $y-2=y-2$

Solution:

identity

Exercise:

Problem: $x+4=x-3$

Solution:

contradiction

Exercise:

Problem: $x+x+x=3x$

Solution:

identity

Exercise:

Problem: $8x = 0$

Solution:

conditional, $x = 0$

Exercise:

Problem: $m - 7 = -5$

Solution:

conditional, $m = 2$

Literal Equations

Literal Equations
Some equations involve more than one variable. Such equations are called **literal equations**.

An equation is solved for a particular variable if that variable alone equals an expression that does not contain that particular variable.
The following equations are examples of literal equations.

1. $y = 2x + 7$. It is solved for y.
2. $d = rt$. It is solved for d.
3. $I = prt$. It is solved for I.
4. $z = \frac{x-u}{s}$. It is solved for z.
5. $y + 1 = x + 4$. This equation is not solved for any particular variable since no variable is isolated.

Solving Equation of the form $x + a = b$ and $x - a = b$

Recall that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side.

$$\begin{array}{ccc} \text{This} & \text{is the} & \text{this} \\ \text{number} & \text{same as} & \text{number} \\ \downarrow & \downarrow & \downarrow \\ x & = & 6 \\ x+2 & = & 8 \\ x-1 & = & 5 \end{array}$$

This suggests the following procedures:

1. We can obtain an equivalent equation (an equation having the same solutions as the original equation) by **adding** the **same number** to **both sides** of the equation.
2. We can obtain an equivalent equation by **subtracting** the **same number** from **both sides** of the equation.

We can use these results to isolate x, thus solving for x.

Example:
Solving $x + a = b$ for x

$$\begin{array}{rcll} x + a & = & b & \text{The } a \text{ is associated with } x \text{ by addition. Undo the association} \\ x + a - a & = & b - a & \text{by subtracting } a \text{ from } both \text{ sides.} \\ x + 0 & = & b - a & a - a = 0 \text{ and } 0 \text{ is the additive identity. } x + 0 = x. \\ x & = & b - a & \text{This equation is equivalent to the first equation, and it is} \\ & & & \text{solved for } x. \end{array}$$

Example:
Solving $x - a = b$ for x

$$\begin{array}{rcll} x - a & = & b & \text{The } a \text{ is associated with } x \text{ by subtraction. Undo the association} \\ x - a + a & = & b + a & \text{by adding } a \text{ to } both \text{ sides.} \\ x + 0 & = & b + a & -a + a = 0 \text{ and } 0 \text{ is the additive identity. } x + 0 = x. \\ x & = & b + a & \text{This equation is equivalent to the first equation, and it is} \\ & & & \text{solved for } x. \end{array}$$

Example:
Method for Solving $x + a = b$ and $x - a = b$ for x
To solve the equation $x + a = b$ for x, **subtract** a from **both** sides of the equation.
To solve the equation $x - a = b$ for x, **add** a to **both** sides of the equation.

Sample Set B

Example:
Solve $x + 7 = 10$ for x.

$$\begin{aligned} x+7 &= 10 && \text{7 is associated with } x \text{ by addition. Undo the association} \\ x+7-7 &= 10-7 && \text{by subtracting 7 from } \textit{both} \text{ sides.} \\ x+0 &= 3 && 7-7=0 \text{ and 0 is the additive identity. } x+0=x. \\ x &= 3 && x \text{ is isolated, and the equation } x=3 \text{ is equivalent to the} \\ & && \text{original equation } x+7=10. \text{ Therefore, these two} \\ & && \text{equation have the same solution. The solution to } x=3 \\ & && \text{is clearly 3. Thus, the solution to } x+7=10 \text{ is also 3.} \end{aligned}$$

Check: Substitute 3 for x in the original equation.

$$\begin{aligned} x+7 &= 10 \\ 3+7 &= 10 && \text{Is this correct?} \\ 10 &= 10 && \text{Yes, this is correct.} \end{aligned}$$

Example:

Solve $m-2=-9$ for m.

$$\begin{aligned} m-2 &= -9 && \text{2 is associated with } m \text{ by subtraction. Undo the association} \\ m-2+2 &= -9+2 && \text{by adding 2 from } \textit{both} \text{ sides.} \\ m+0 &= -7 && -2+2=0 \text{ and 0 is the additive identity. } m+0=m. \\ m &= -7 \end{aligned}$$

Check: Substitute -7 for m in the original equation.

$$\begin{aligned} m-2 &= -9 \\ -7-2 &= -9 && \text{Is this correct?} \\ -9 &= -9 && \text{Yes, this is correct.} \end{aligned}$$

Example:

Solve $y-2.181=-16.915$ for y.

$$\begin{aligned} y-2.181 &= -16.915 \\ y-2.181+2.181 &= -16.915+2.181 \\ y &= -14.734 \end{aligned}$$

On the Calculator

Type	16.915
Press	$\boxed{+/-}$
Press	$\boxed{+}$
Type	2.181
Press	$\boxed{=}$
Display reads:	-14.734

Example:

Solve $y+m=s$ for y.

$$\begin{aligned} y+m &= s && m \text{ is associated with } y \text{ by addition. Undo the association} \\ y+m-m &= s-m && \text{by subtracting } m \text{ from } \textit{both} \text{ sides.} \\ y+0 &= s-m && m-m=0 \text{ and } 0 \text{ is the additive identity. } y+0=y. \\ y &= s-m \end{aligned}$$

Check: Substitute $s-m$ for y in the original equation.

$$\begin{aligned} y+m &= s \\ s-m+m &= s && \text{Is this correct?} \\ s &= s \quad \text{True} && \text{Yes, this is correct.} \end{aligned}$$

Example:

Solve $k-3h=-8h+5$ for k.

$$\begin{aligned} k-3h &= -8h+5 && 3h \text{ is associated with } k \text{ by subtraction. Undo the association} \\ k-3h+3h &= -8h+5+3h && \text{by adding } 3h \text{ to } \textit{both} \text{ sides.} \\ k+0 &= -5h+5 && -3h+3h=0 \text{ and } 0 \text{ is the additive identity. } k+0=k. \\ k &= -5h+5 \end{aligned}$$

Practice Set B

Exercise:

Problem: Solve $y-3=8$ for y.

Solution:

$y=11$

Exercise:

Problem: Solve $x + 9 = -4$ for x.

Solution:

$x = -13$

Exercise:

Problem: Solve $m + 6 = 0$ for m.

Solution:

$m = -6$

Exercise:

Problem: Solve $g - 7.2 = 1.3$ for g.

Solution:

$g = 8.5$

Exercise:

Problem: solve $f + 2d = 5d$ for f.

Solution:

$f = 3d$

Exercise:

Problem: Solve $x + 8y = 2y - 1$ for x.

Solution:

$x = -6y - 1$

Exercise:

Problem: Solve $y + 4x - 1 = 5x + 8$ for y.

Solution:

$y = x + 9$

Exercises

For the following problems, classify each of the equations as an identity, contradiction, or conditional equation.

Exercise:

Problem: $m + 6 = 15$

Solution:

conditional

Exercise:

Problem: $y - 8 = -12$

Exercise:

Problem: $x + 1 = x + 1$

Solution:

identity

Exercise:

Problem: $k - 2 = k - 3$

Exercise:

Problem: $g + g + g + g = 4g$

Solution:

identity

Exercise:

Problem: $x + 1 = 0$

For the following problems, determine which of the literal equations have been solved for a variable. Write "solved" or "not solved."

Exercise:

Problem: $y = 3x + 7$

Solution:

solved

Exercise:

Problem: $m = 2k + n - 1$

Exercise:

Problem: $4a = y - 6$

Solution:

not solved

Exercise:

Problem: $hk = 2k + h$

Exercise:

Problem: $2a = a + 1$

Solution:

not solved

Exercise:

Problem: $5m = 2m - 7$

Exercise:

Problem: $m = m$

Solution:

not solved

For the following problems, solve each of the conditional equations.

Exercise:

Problem: $h - 8 = 14$

Exercise:

Problem: $k + 10 = 1$

Solution:

$k = -9$

Exercise:

Problem: $m - 2 = 5$

Exercise:

Problem: $y + 6 = -11$

Solution:

$y = -17$

Exercise:

Problem: $y - 8 = -1$

Exercise:

Problem: $x + 14 = 0$

Solution:

$x = -14$

Exercise:

Problem: $m - 12 = 0$

Exercise:

Problem: $g + 164 = -123$

Solution:

$g = -287$

Exercise:

Problem: $h - 265 = -547$

Exercise:

Problem: $x + 17 = -426$

Solution:

$x = -443$

Exercise:

Problem: $h - 4.82 = -3.56$

Exercise:

Problem: $y + 17.003 = -1.056$

Solution:

$y = -18.059$

Exercise:

Problem: $k + 1.0135 = -6.0032$

Exercise:

Problem: Solve $n + m = 4$ for n.

Solution:

$n = 4 - m$

Exercise:

Problem: Solve $P + 3Q - 8 = 0$ for P.

Exercise:

Problem: Solve $a + b - 3c = d - 2f$ for b.

Solution:

$b = -a + 3c + d - 2f$

Exercise:

Problem: Solve $x - 3y + 5z + 1 = 2y - 7z + 8$ for x.

Exercise:

Problem: Solve $4a - 2b + c + 11 = 6a - 5b$ for c.

Solution:

$c = 2a - 3b - 11$

Exercises for Review

Exercise:

Problem: ([link]) Simplify $(4x^5y^2)^3$.

Exercise:

Problem: ([link]) Write $\frac{20x^3y^7}{5x^5y^3}$ so that only positive exponents appear.

Solution:

$\frac{4y^4}{x^2}$

Exercise:

Problem:

([link]) Write the number of terms that appear in the expression $5x^2 + 2x - 6 + (a + b)$, and then list them.

Exercise:

Problem: **([link])** Find the product. $(3x - 1)^2$.

Solution:

$9x^2 - 6x + 1$

Exercise:

Problem: **([link])** Specify the domain of the equation $y = \frac{5}{x-2}$.

Solving Linear Equations and Inequalities
<para>This module is from <link document="col10614">Elementary Algebra</link> by Denny Burzynski and Wade Ellis, Jr.</para> This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter, the emphasis is on the mechanics of equation solving, which clearly explains how to isolate a variable. The goal is to help the student feel more comfortable with solving applied problems. Ample opportunity is provided for the student to practice translating words to symbols, which is an important part of the "Five-Step Method" of solving applied problems (discussed in modules (<link document="m21980"/>) and (<link document="m21979"/>)). Objectives of this module: be able to identify the solution of a linear equation in two variables, know that solutions to linear equations in two variables can be written as ordered pairs.

Overview

- Solutions to Linear Equations in Two Variables
- Ordered Pairs as Solutions

Solutions to Linear Equations in Two Variables

Solution to an Equation in Two Variables
We have discovered that an equation is a mathematical way of expressing the relationship of equality between quantities. If the relationship is between two quantities, the equation will contain two variables. We say that an equation in two variables has a solution if an ordered **pair** of values can be found such that when these two values are substituted into the equation a true statement results. This is illustrated when we observe some solutions to the equation $y = 2x + 5$.

1. $x = 4,\ y = 13; \qquad \text{since } 13 = 2(4) + 5 \text{ is true.}$
2. $x = 1,\ y = 7; \qquad \text{since } 7 = 2(1) + 5 \text{ is true.}$
3. $x = 0,\ y = 5; \qquad \text{since } 5 = 2(0) + 5 \text{ is true.}$
4. $x = -6,\ y = -7; \qquad \text{since } -7 = 2(-6) + 5 \text{ is true.}$

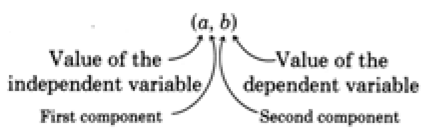
Ordered Pairs as Solutions

It is important to keep in mind that a solution to a linear equation in two variables is an ordered pair of values, one value for each variable. A solution is not completely known until the values of **both** variables are specified.

Independent and Dependent Variables
Recall that, in an equation, any variable whose value can be freely assigned is said to be an **independent variable.** Any variable whose value is determined once the other value or values have been assigned is said to be a **dependent variable.** If, in a linear equation, the independent variable is x and the dependent variable is y, and a solution to the equation is $x = a$ and $y = b$, the solution is written as the

ORDERED PAIR (a, b)

Ordered Pair
In an **ordered pair**, (a, b), the first component, a, gives the value of the independent variable, and the second component, b, gives the value of the dependent variable.



We can use ordered pairs to show some solutions to the equation $y = 6x - 7$.

Example:
$(0, -7)$.
If $x = 0$ and $y = -7$, we get a true statement upon substitution and computataion.

$$\begin{aligned} y &= 6x-7 & \\ -7 &= 6(0)-7 & \text{Is this correct?} \\ -7 &= -7 & \text{Yes, this is correct.} \end{aligned}$$

Example:
$(8,\ 41)$.
If $x=8$ and $y=41$, we get a true statement upon substitution and computataion.

$$\begin{aligned} y &= 6x-7 & \\ 41 &= 6(8)-7 & \text{Is this correct?} \\ 41 &= 48-7 & \text{Is this correct?} \\ 41 &= 41 & \text{Yes, this is correct.} \end{aligned}$$

Example:
$(-4,\ -31)$.
If $x=-4$ and $y=-31$, we get a true statement upon substitution and computataion.

$$\begin{aligned} y &= 6x-7 & \\ -31 &= 6(-4)-7 & \text{Is this correct?} \\ -31 &= -24-7 & \text{Is this correct?} \\ -31 &= -31 & \text{Yes, this is correct.} \end{aligned}$$

These are only three of the infintely many solutions to this equation.

Sample Set A

Find a solution to each of the following linear equations in two variables and write the solution as an ordered pair.

Example:
$y = 3x - 6$, if $x = 1$
Substitute 1 for x, compute, and solve for y.
$$\begin{aligned} y &= 3(1) - 6 \\ &= 3 - 6 \\ &= -3 \end{aligned}$$
Hence, one solution is $(1,\ -3)$.

Example:
$y = 15 - 4x$, if $x = -10$
Substitute -10 for x, compute, and solve for y.
$$\begin{aligned} y &= 15 - 4(-10) \\ &= 15 + 40 \\ &= 55 \end{aligned}$$
Hence, one solution is $(-10,\ 55)$.

Example:
$b = -9a + 21$, if $a = 2$
Substitute 2 for a, compute, and solve for b.
$$\begin{aligned} b &= -9(2) + 21 \\ &= -18 + 21 \\ &= 3 \end{aligned}$$
Hence, one solution is $(2,\ 3)$.

Example:

$5x-2y=1$, if $x=0$
Substitute 0 for x, compute, and solve for y.

$$\begin{aligned} 5(0)-2y &= 1 \\ 0-2y &= 1 \\ -2y &= 1 \\ y &= -\frac{1}{2} \end{aligned}$$

Hence, one solution is $\left(0,\ -\frac{1}{2}\right)$.

Practice Set A

Find a solution to each of the following linear equations in two variables and write the solution as an ordered pair.

Exercise:

Problem: $y=7x-20$, if $x=3$

Solution:

$(3,\ 1)$

Exercise:

Problem: $m=-6n+1$, if $n=2$

Solution:

$(2,\ -11)$

Exercise:

Problem: $b=3a-7$, if $a=0$

Solution:

$(0,\ -7)$

Exercise:

Problem: $10x - 5y - 20 = 0, \text{ if } x = -8$

Solution:

$(-8,\ -20)$

Exercise:

Problem: $3a + 2b + 6 = 0, \text{ if } a = -1$

Solution:

$\left(-1,\ \frac{-3}{2}\right)$

Exercises

For the following problems, solve the linear equations in two variables.

Exercise:

Problem: $y = 8x + 14, \text{ if } x = 1$

Solution:

$(1, 22)$

Exercise:

Problem: $y = -2x + 1, \text{ if } x = 0$

Exercise:

Problem: $y = 5x + 6, \text{ if } x = 4$

Solution:

$(4, 26)$

Exercise:

Problem: $x + y = 7$, if $x = 8$

Exercise:

Problem: $3x + 4y = 0$, if $x = -3$

Solution:

$\left(-3, \frac{9}{4}\right)$

Exercise:

Problem: $-2x + y = 1$, if $x = \frac{1}{2}$

Exercise:

Problem: $5x - 3y + 1 = 0$, if $x = -6$

Solution:

$\left(-6, -\frac{29}{3}\right)$

Exercise:

Problem: $-4x - 4y = 4$, if $y = 7$

Exercise:

Problem: $2x + 6y = 1$, if $y = 0$

Solution:

$\left(\frac{1}{2}, 0\right)$

Exercise:

Problem: $-x-y=0$, if $y=\frac{14}{3}$

Exercise:

Problem: $y=x$, if $x=1$

Solution:

$(1,1)$

Exercise:

Problem: $x+y=0$, if $x=0$

Exercise:

Problem: $y+\frac{3}{4}=x$, if $x=\frac{9}{4}$

Solution:

$\left(\frac{9}{4},\frac{3}{2}\right)$

Exercise:

Problem: $y+17=x$, if $x=-12$

Exercise:

Problem: $-20y+14x=1$, if $x=8$

Solution:

$\left(8,\frac{111}{20}\right)$

Exercise:

Problem: $\frac{3}{5}y+\frac{1}{4}x=\frac{1}{2}$, if $x=-3$

Exercise:

Problem: $\frac{1}{5}x + y = -9$, if $y = -1$

Solution:

$(-40, -1)$

Exercise:

Problem: $y + 7 - x = 0$, if $x =$

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Exercise:

Problem: $2x + 31y - 3 = 0$, if $x = a$

Solution:

$\left(a, \frac{3-2a}{31}\right)$

Exercise:

Problem: $436x + 189y = 881$, if $x = -4231$

Exercise:

Problem: $y = 6(x - 7)$, if $x = 2$

Solution:

$(2, -30)$

Exercise:

Problem: $y = 2(4x + 5)$, if $x = -1$

Exercise:

Problem: $5y = 9(x - 3)$, if $x = 2$

Solution:

$\left(2, -\frac{9}{5}\right)$

Exercise:

Problem: $3y = 4(4x + 1)$, if $x = -3$

Exercise:

Problem: $-2y = 3(2x - 5)$, if $x = 6$

Solution:

$\left(6, -\frac{21}{2}\right)$

Exercise:

Problem: $-8y = 7(8x + 2)$, if $x = 0$

Exercise:

Problem: $b = 4a - 12$, if $a = -7$

Solution:

$(-7, -40)$

Exercise:

Problem: $b = -5a + 21$, if $a = -9$

Exercise:

Problem: $4b - 6 = 2a + 1$, if $a = 0$

Solution:

$\left(0, \frac{7}{4}\right)$

Exercise:

Problem: $-5m + 11 = n + 1$, if $n = 4$

Exercise:

Problem: $3(t + 2) = 4(s - 9)$, if $s = 1$

Solution:

$\left(1, -\frac{38}{3}\right)$

Exercise:

Problem: $7(t - 6) = 10(2 - s)$, if $s = 5$

Exercise:

Problem: $y = 0x + 5$, if $x = 1$

Solution:

$(1, 5)$

Exercise:

Problem: $2y = 0x - 11$, if $x = -7$

Exercise:

Problem: $-y = 0x + 10$, if $x = 3$

Solution:

$(3, -10)$

Exercise:

Problem: $-5y = 0x - 1$, if $x = 0$

Exercise:

Problem: $y = 0(x - 1) + 6$, if $x = 1$

Solution:

$(1, 6)$

Exercise:

Problem: $y = 0(3x + 9) - 1$, if $x = 12$

Calculator Problems

Exercise:

Problem:

An examination of the winning speeds in the Indianapolis 500 automobile race from 1961 to 1970 produces the equation $y = 1.93x + 137.60$, where x is the number of years from 1960 and y is the winning speed. Statistical methods were used to obtain the equation, and, for a given year, the equation gives only the approximate winning speed. Use the equation $y = 1.93x + 137.60$ to find the approximate winning speed in

a. 1965
b. 1970

c. 1986
d. 1990

Solution:

(a) Approximately 147 mph using $(5, 147.25)$
(b) Approximately 157 mph using $(10, 156.9)$
(c) Approximately 188 mph using $(26, 187.78)$
(d) Approximately 196 mph using $(30, 195.5)$

Exercise:

Problem:

In electricity theory, Ohm's law relates electrical current to voltage by the equation $y = 0.00082x$, where x is the voltage in volts and y is the current in amperes. This equation was found by statistical methods and for a given voltage yields only an approximate value for the current. Use the equation $y = 0.00082x$ to find the approximate current for a voltage of

a. 6 volts
b. 10 volts

Exercise:

Problem:

Statistical methods have been used to obtain a relationship between the actual and reported number of German submarines sunk each month by the U.S. Navy in World War II. The equation expressing the approximate number of actual sinkings, y, for a given number of reported sinkings, x, is $y = 1.04x + 0.76$. Find the approximate number of actual sinkings of German submarines if the reported number of sinkings is

a. 4
b. 9

c. 10

Solution:

(a) Approximately 5 sinkings using $(4, 4.92)$
(b) Approximately 10 sinkings using $(9, 10.12)$
(c) Approximately 11 sinkings using $(10, 11.16)$

Exercise:

Problem:

Statistical methods have been used to obtain a relationship between the heart weight (in milligrams) and the body weight (in milligrams) of 10-month-old diabetic offspring of crossbred male mice. The equation expressing the approximate body weight for a given heart weight is $y = 0.213x - 4.44$. Find the approximate body weight for a heart weight of

a. 210 mg
b. 245 mg

Exercise:

Problem:

Statistical methods have been used to produce the equation $y = 0.176x - 0.64$. This equation gives the approximate red blood cell count (in millions) of a dog's blood, y, for a given packed cell volume (in millimeters), x. Find the approximate red blood cell count for a packed cell volume of

a. 40 mm
b. 42 mm

Solution:

(a) Approximately 6.4 using $(40, 6.4)$
(b) Approximately 4.752 using $(42, 7.752)$

Exercise:

Problem:

An industrial machine can run at different speeds. The machine also produces defective items, and the number of defective items it produces appears to be related to the speed at which the machine is running. Statistical methods found that the equation $y = 0.73x - 0.86$ is able to give the approximate number of defective items, y, for a given machine speed, x. Use this equation to find the approximate number of defective items for a machine speed of

a. 9
b. 12

Exercise:

Problem:

A computer company has found, using statistical techniques, that there is a relationship between the aptitude test scores of assembly line workers and their productivity. Using data accumulated over a period of time, the equation $y = 0.89x - 41.78$ was derived. The x represents an aptitude test score and y the approximate corresponding number of items assembled per hour. Estimate the number of items produced by a worker with an aptitude score of

a. 80
b. 95

Solution:

(a) Approximately 29 items using $(80, 29.42)$
(b) Approximately 43 items using $(95, 42.77)$

Exercise:

Problem:

Chemists, making use of statistical techniques, have been able to express the approximate weight of potassium bromide, W, that will dissolve in 100 grams of water at T degrees centigrade. The equation expressing this relationship is $W = 0.52T + 54.2$. Use this equation to predict the potassium bromide weight that will dissolve in 100 grams of water that is heated to a temperature of

a. 70 degrees centigrade
b. 95 degrees centigrade

Exercise:

Problem:

The marketing department at a large company has been able to express the relationship between the demand for a product and its price by using statistical techniques. The department found, by analyzing studies done in six different market areas, that the equation giving the approximate demand for a product (in thousands of units) for a particular price (in cents) is $y = -14.15x + 257.11$. Find the approximate number of units demanded when the price is

a. $\$0.12$
b. $\$0.15$

Solution:

(a) Approximately 87 units using $(12, 87.31)$
(b) Approximately 45 units using $(15, 44.86)$

Exercise:

Problem:

The management of a speed-reading program claims that the approximate speed gain (in words per minute), G, is related to the number of weeks spent in its program, W, is given by the equation $G = 26.68W - 7.44$. Predict the approximate speed gain for a student who has spent

a. 3 weeks in the program
b. 10 weeks in the program

Exercises for Review

Exercise:

Problem: (**[link]**) Find the product. $(4x - 1)(3x + 5)$.

Solution:

$12x^2 + 17x - 5$

Exercise:

Problem: (**[link]**) Find the product. $(5x + 2)(5x - 2)$.

Exercise:

Problem: (**[link]**) Solve the equation $6[2(x - 4) + 1] = 3[2(x - 7)]$.

Solution:

$x = 0$

Exercise:

Problem: (**[link]**) Solve the inequality $-3a - (a - 5) \geq a + 10$.

Exercise:

Problem:

([link]) Solve the compound inequality $-1 < 4y + 11 < 27$.

Solution:

$-3 < y < 4$

Introduction to Graphing
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: be familiar with the plane, know what is meant by the coordinates of a point, be able to plot points in the plane.

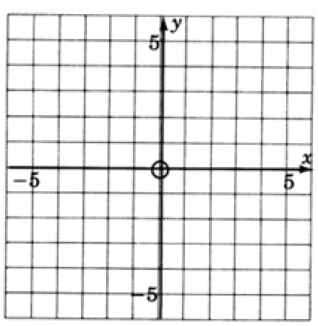
Overview

- The Plane
- Coordinates of a Point
- Plotting Points

The Plane

Ordered Pairs
We are now interested in studying graphs of linear equations in two variables. We know that solutions to equations in two variables consist of a pair of values, one value for each variable. We have called these pairs of values **ordered pairs.** Since we have a pair of values to graph, we must have a pair of axes (number lines) upon which the values can be located.

Origin
We draw the axes so they are perpendicular to each other and so that they intersect each other at their $0'$s. This point is called the **origin.**

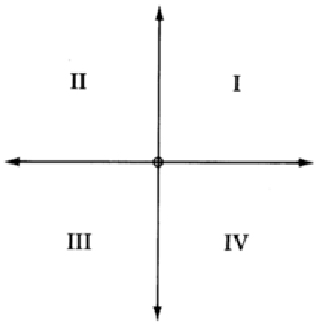


Rectangular Coordinate System
These two lines form what is called a **rectangular coordinate system.** They also determine a plane.

xy-plane
A **plane** is a flat surface, and a result from geometry states that through any two intersecting lines (the axes) exactly one plane (flat surface) may be passed. If we are dealing with a linear equation in the two

variables x and y, we sometimes say we are graphing the equation using a rectangular coordinate system, or that we are graphing the equation in the xy-plane.

Quadrant
Notice that the two intersecting coordinate axes divide the plane into four equal regions. Since there are four regions, we call each one a **quadrant** and number them counterclockwise using Roman numerals.



Recall that when we first studied the number line we observed the following:

For each real number there exists a unique point on the number line, and for each point on the number line we can associate a unique real number.

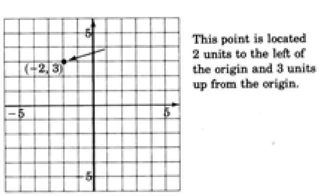
We have a similar situation for the plane.

For each ordered pair $(a,\ b)$, there exists a unique point in the plane, and to each point in the plane we can associate a unique ordered pair $(a,\ b)$ of real numbers.

Coordinates of a Point

Coordinates of a Point
The numbers in an ordered pair that are associated with a particular point are called the **coordinates of the point.** The **first number** in the ordered pair expresses the point's horizontal distance and direction (left or right) from the origin. The **second number** expresses the point's vertical distance and direction (up or down) from the origin.

The Coordinates Determine Distance and Direction
A **positive number** means a direction to the **right or up**. A **negative number** means a direction to the **left or down**.



Plotting Points

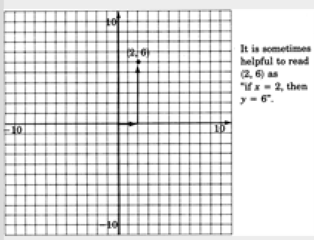
Since points and ordered pairs are so closely related, the two terms are sometimes used interchangeably. The following two phrases have the same meaning:

1. Plot the point $(a,\ b)$.
2. Plot the ordered pair $(a,\ b)$.

Plotting a Point
Both phrases mean: Locate, in the plane, the point associated with the ordered pair $(a,\ b)$ and draw a mark at that position.

Sample Set A

Example:
Plot the ordered pair $(2,\ 6)$.
We begin at the origin. The first number in the ordered pair, 2, tells us we move 2 units to the right ($+2$ means 2 units to the right) The second number in the ordered pair, 6, tells us we move 6 units up ($+6$ means 6 units up).

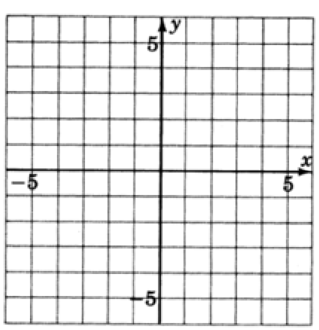


Practice Set A

Exercise:

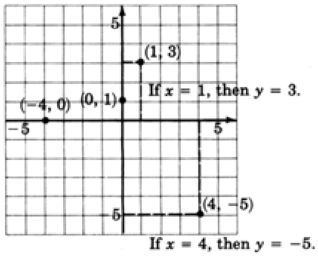
Problem: Plot the ordered pairs.

$(1,\ 3),\ (4,\ -5),\ (0,\ 1),\ (-4,\ 0).$



Solution:

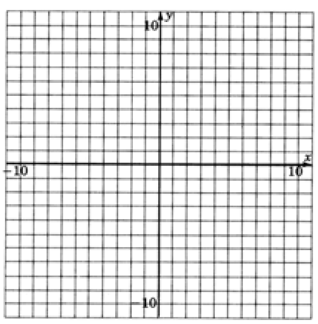
(Notice that the dotted lines on the graph are only for illustration and should not be included when plotting points.)



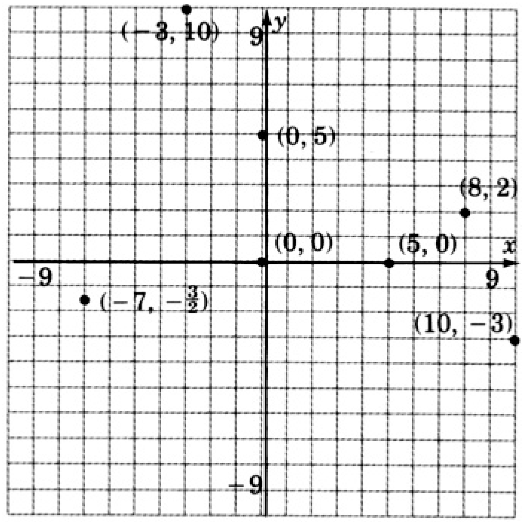
Exercises

Exercise:

Problem: Plot the following ordered pairs. (Do not draw the arrows as in Practice Set A.) $(8,\ 2),\ (10,\ -3),\ (-3,\ 10),\ (0,\ 5),\ (5,\ 0),\ (0,\ 0),\ \left(-7,\ -\frac{3}{2}\right).$



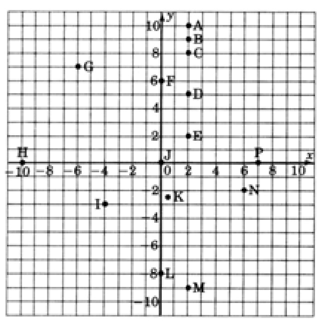
Solution:



Exercise:

Problem:

As accurately as possible, state the coordinates of the points that have been plotted on the following graph.



Exercise:

Problem: Using ordered pair notation, what are the coordinates of the origin?

Solution:

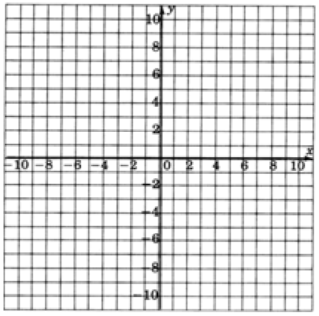
$\text{Coordinates of the origin are } (0, 0).$

Exercise:

Problem:

We know that solutions to linear equations in two variables can be expressed as ordered pairs. Hence, the solutions can be represented as points in the plane. Consider the linear equation $y = 2x - 1$. Find at least ten solutions to this equation by choosing x-values between -4 and 5 and computing the corresponding y-values. Plot these solutions on the coordinate system below. Fill in the table to help you keep track of the ordered pairs.

x												
y												



Keeping in mind that there are infinitely many ordered pair solutions to $y = 2x - 1$, speculate on the geometric structure of the graph of all the solutions. Complete the following statement:

The name of the type of geometric structure of the graph of all the solutions to the linear equation $y = 2x - 1$ seems to be ___________ .

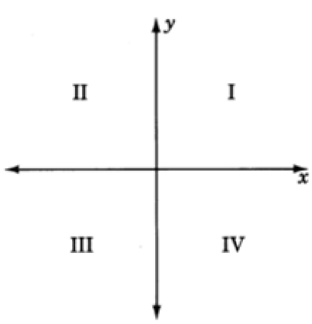
Where does this figure cross the y-axis? Does this number appear in the equation $y = 2x - 1$?

Place your pencil at any point on the figure (you may have to connect the dots to see the figure clearly). Move your pencil exactly one unit to the right (horizontally). To get back onto the figure, you must move your pencil either up or down a particular number of units. How many units must you move vertically to get back onto the figure, and do you see this number in the equation $y = 2x - 1$?

Exercise:

Consider the xy-plane.

Problem:



Complete the table by writing the appropriate inequalities.

I	**II**	**III**	**IV**
$x > 0$	$x < 0$	x	x
$y > 0$	y	y	y

In the following problems, the graphs of points are called **scatter diagrams** and are frequently used by statisticians to determine if there is a relationship between the two variables under consideration. The first component of the ordered pair is called the **input variable** and the second component is called the **output variable**. Construct the scatter diagrams. Determine if there appears to be a relationship between the two variables under consideration by making the following observations: A relationship may exist if

a. as one variable increases, the other variable increases
b. as one variable increases, the other variable decreases

Solution:

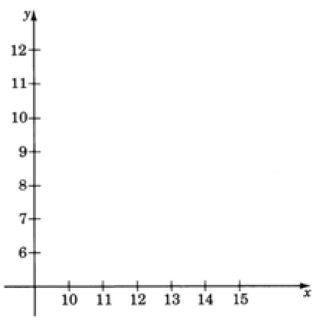
I	**II**	**III**	**IV**
$x > 0$	$x < 0$	$x < 0$	$x > 0$
$y > 0$	$y > 0$	$y < 0$	$y < 0$

Exercise:

Problem:

A psychologist, studying the effects of a placebo on assembly line workers at a particular industrial site, noted the time it took to assemble a certain item before the subject was given the placebo, x, and the time it took to assemble a similar item after the subject was given the placebo, y. The psychologist's data are

x	y
10	8
12	9
11	9
10	7
14	11
15	12
13	10



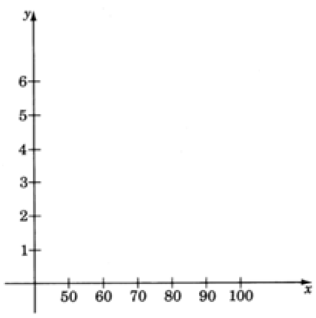
Exercise:

Problem:

The following data were obtained in an engineer's study of the relationship between the amount of pressure used to form a piece of machinery, x, and the number of defective pieces of machinery produced, y.

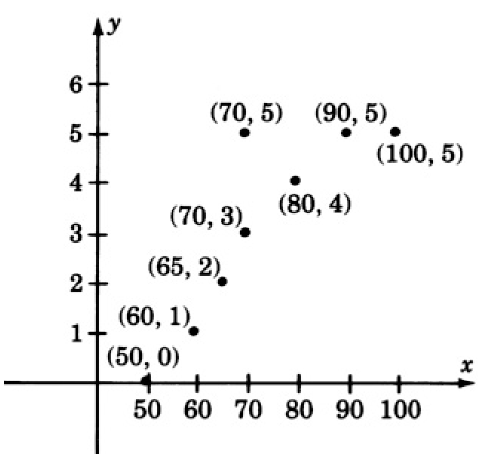
x	y
50	0
60	1
65	2

70	3
80	4
70	5
90	5
100	5



Solution:

Yes, there does appear to be a relation.

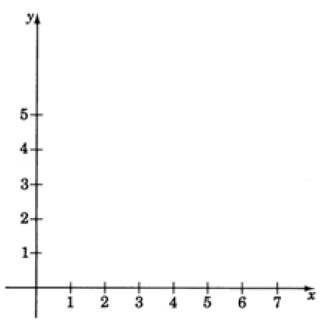


Exercise:

Problem:

The following data represent the number of work days missed per year, x, by the employees of an insurance company and the number of minutes they arrive late from lunch, y.

x	y
1	3
6	4
2	2
2	3
3	1
1	4
4	4
6	3
5	2
6	1



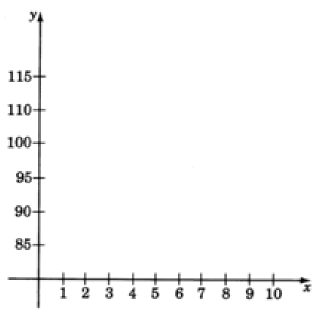
Exercise:

Problem:

A manufacturer of dental equipment has the following data on the unit cost (in dollars), y, of a particular item and the number of units, x, manufactured for each order.

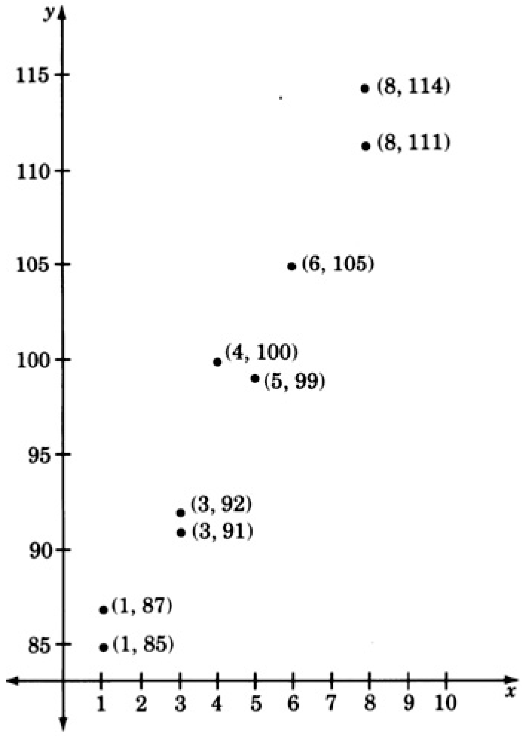
x	y

1	85
3	92
5	99
3	91
4	100
1	87
6	105
8	111
8	114



Solution:

Yes, there does appear to be a relation.



Exercises for Review

Exercise:

Problem: ([link]) Simplify $\left(\frac{18x^5y^6}{9x^2y^4}\right)^5$.

Exercise:

Problem:

([link]) Supply the missing word. An is a statement that two algebraic expressions are equal.

Solution:

equation

Exercise:

Problem: ([link]) Simplify the expression $5xy(xy - 2x + 3y) - 2xy(3xy - 4x) - 15xy^2$.

Exercise:

Problem:

([link]) Identify the equation $x + 2 = x + 1$ as an identity, a contradiction, or a conditional equation.

Solution:

contradiction

Exercise:

Problem:

([link]) Supply the missing phrase. A system of axes constructed for graphing an equation is called a .

Finding the Equation of a Line
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: be able to find the equation of a line using either the slope-intercept form or the point-slope form of a line.

Overview

- The Slope-Intercept and Point-Slope Forms

The Slope-Intercept and Point-Slope Forms

In the pervious sections we have been given an equation and have constructed the line to which it corresponds. Now, however, suppose we're given some geometric information about the line and we wish to construct the corresponding equation. We wish to find the equation of a line.

We know that the formula for the slope of a line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.We can find the equation of a line using the slope formula in either of two ways:

Example:
If we’re given the slope, m, and **any** point (x_1, y_1) on the line, we can substitute this information into the formula for slope.
Let (x_1, y_1) be the known point on the line and let (x, y) be any other point on the line. Then

$$\begin{aligned}
m &= \frac{y-y_1}{x-x_1} && \text{Multiply both sides by} x - x_1. \\
m(x-x_1) &= \cancel{(x-x_1)} \cdot \frac{y-y_1}{\cancel{x-x_1}} && \\
m(x-x_1) &= y - y_1 && \text{For convenience, we'll rewrite the equation.} \\
y - y_1 &= m(x-x_1)
\end{aligned}$$

Since this equation was derived using a point and the slope of a line, it is called the **point-slope** form of a line.

Example:

If we are given the slope, m, y-intercept , $(0, b)$, we can substitute this information into the formula for slope.

Let $(0, b)$ be the y-intercept and (x, y) be any other point on the line. Then,

$$\begin{aligned}
m &= \frac{y-b}{x-0} && \\
m &= \frac{y-b}{x} && \text{Multiply both sides by } x. \\
m \cdot x &= \cancel{x} \cdot \frac{y-b}{\cancel{x}} && \\
mx &= y - b && \text{Solve for } y. \\
mx + b &= y && \text{For convenience, we'll rewrite this equation.} \\
y &= mx + b
\end{aligned}$$

Since this equation was derived using the slope and the intercept, it was called the **slope-intercept** form of a line.

We summarize these two derivations as follows.

Forms of the Equation of a Line

We can find the equation of a line if we're given either of the following sets of information:

1. The slope, m,and the y-intercept, $(0, b)$,by substituting these values into

   $$\boxed{y = mx + b}$$

   This is the slope-intercept form.
2. The slope, m,and any point, (x_1, y_1),by substituting these values into

   $$\boxed{y - y_1 = m(x - x_1)}$$

This is the point-slope form.

Notice that both forms rely on knowing the slope. If we are given two points on the line we may still find the equation of the line passing through them by first finding the slope of the line, then using the point-slope form.

It is customary to use either the slope-intercept form or the general form for the final form of the line. We will use the slope-intercept form as the final form.

Sample Set A

Find the equation of the line using the given information.

Example:
$m=6$, y-intercept $(0,4)$
Since we're given the slope and the y-intercept, we'll use the slope-intercept form.
$m=6, b=4.$
$y=mx+b$
$y=6x+4$

Example:
$m=-\frac{3}{4}$, y-intercept $\left(0,\frac{1}{8}\right)$
Since we're given the slope and the y-intercept, we'll use the slope-intercept form.
$m=\frac{-3}{4},$

$$\begin{aligned} b &= \frac{1}{8}. \\ y &= mx+b \\ y &= -\frac{3}{4}x+\frac{1}{8} \end{aligned}$$

Example:
$m=2,$ the point $(4,3).$
Write the equation in slope-intercept form.
Since we're given the slope and some point, we'll use the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Let } (x_1, y_1\ ) \text{ be (4,3).} \\ y - 3 &= 2(x - 4) && \text{Put this equation in slope-intercept form by solving for } y. \\ y - 3 &= 2x - 8 \\ y &= 2x - 5 \end{aligned}$$

Example:

$m = -5, \quad \text{the point}(-3, 0).$

Write the equation in slope-intercept form.

Since we're given the slope and some point, we'll use the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Let } (x_1, y_1\ ) \text{ be (-3,0).} \\ y - 0 &= -5\left[x - (-3)\right] \\ y &= -5(x + 3) && \text{Solve for } y. \\ y &= -5x - 15 \end{aligned}$$

Example:

$m = -1, \quad \text{the point}(0, 7).$

Write the equation in slope-intercept form.

We're given the slope and a point, but careful observation reveals that this point is actually the y-intercept. Thus, we'll use the slope-intercept form. If we had not seen that this point was the y-interceptwe would have proceeded with the point-slope form. This would create slightly more work, but still give the same result.

Slope-intercept form Point-slope form

$$\begin{aligned} y &= mx + b \\ y &= -1x + 7 \\ y &= -x + 7 \end{aligned} \qquad \begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= -1(x - 0) \\ y - 7 &= -x \\ y &= -x + 7 \end{aligned}$$

Example:

The two points $(4, 1)$and $(3, 5)$.

Write the equation in slope-intercept form.

Since we're given two points, we'll find the slope first.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{3-4} = \frac{4}{-1} = -4$

Now, we have the slope and two points. We can use either point and the point-slope form.

Using $(4\ ,\ 1)$	**Using** $(3\ ,\ 5)$
$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-1 &= -4(x-4) \\ y-1 &= -4x+16 \\ y &= -4x+17 \end{aligned}$	$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-5 &= -4(x-3) \\ y-5 &= -4x+12 \\ y &= -4x+17 \end{aligned}$
We can see that the use of either point gives the same result.	

Practice Set A

Find the equation of each line given the following information. Use the slope-intercept form as the final form of the equation.

Exercise:

Problem: $m = 5\ ,\ y$-intercept $(0, 8)$.

Solution:

$y = 5x + 8$

Exercise:

Problem: $m = -8\ ,\ y$-intercept $(0, 3)$.

Solution:

$y = -8x + 3$

Exercise:

Problem: $m = 2\ ,\ y$-intercept $(0, -7)$.

Solution:

$y = 2x - 7$

Exercise:

Problem: $m = 1\ ,\ y$-intercept $(0, -1)$.

Solution:

$y=x-1$

Exercise:

Problem: $m=-1$, y-intercept $(0,-10)$.

Solution:

$y=-x-10$

Exercise:

Problem: $m=4$,the point $(5,2)$.

Solution:

$y=4x-18$

Exercise:

Problem: $m=-6$,the point $(-1,0)$.

Solution:

$y=-6x-6$

Exercise:

Problem: $m=-1$,the point $(-5,-5)$.

Solution:

$y=-x-10$

Exercise:

Problem: The two points $(4,1)$and $(6,5)$.

Solution:

$y=2x-7$

Exercise:

Problem: The two points $(-7,-1)$and $(-4,8)$.

Solution:

$y=3x+20$

Sample Set B

Example:
Find the equation of the line passing through the point $(4,-7)$having slope 0.
We're given the slope and some point, so we'll use the point-slope form. With $m=0$ and (x_1,y_1)as $(4,-7)$,we have

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-7) &= 0(x-4) \\ y+7 &= 0 \\ y &= -7 \end{aligned}$$

This is a horizontal line.

Example:
Find the equation of the line passing through the point $(1,3)$given that the line is vertical.
Since the line is vertical, the slope does not exist. Thus, we cannot use either the slope-intercept form or the point-slope form. We must recall what we know about vertical lines. The equation of this line is simply $x=1$.

Practice Set B

Exercise:

Problem:

Find the equation of the line passing through the point $(-2,9)$having slope 0.

Solution:

$y=9$

Exercise:

Problem:

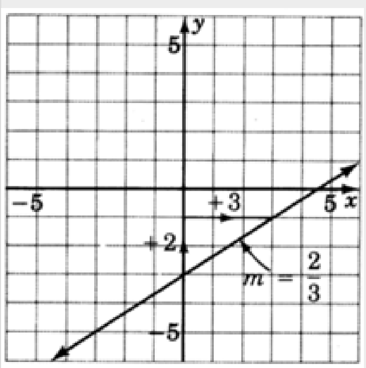
Find the equation of the line passing through the point $(-1,6)$given that the line is vertical.

Solution:

$x=-1$

Sample Set C

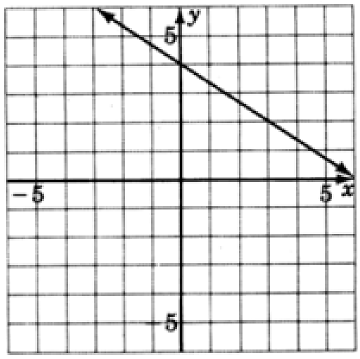
Example:
Reading only from the graph, determine the equation of the line.
The slope of the line is $\frac{2}{3}$, and the line crosses the y-axisat the point $(0,-3)$.Using the slope-intercept form we get
$y=\frac{2}{3}x-3$



Practice Set C

Exercise:

Problem: Reading only from the graph, determine the equation of the line.



Solution:

$y = \frac{-2}{3}x + 4$

Exercises

For the following problems, write the equation of the line using the given information in slope-intercept form.

Exercise:

Problem: $m = 3$, y-intercept $(0, 4)$

Solution:

$y = 3x + 4$

Exercise:

Problem: $m = 2$, y-intercept $(0, 5)$

Exercise:

Problem: $m = 8$, y-intercept $(0, 1)$

Solution:

$y = 8x + 1$

Exercise:

Problem: $m = 5$, y-intercept $(0, -3)$

Exercise:

Problem: $m = -6$, y-intercept $(0, -1)$

Solution:

$y = -6x - 1$

Exercise:

Problem: $m = -4$, y-intercept $(0, 0)$

Exercise:

Problem: $m = -\frac{3}{2}$, y-intercept $(0, 0)$

Solution:

$y = -\frac{3}{2}x$

Exercise:

Problem: $m = 3$, $(1, 4)$

Exercise:

Problem: $m = 1$, $(3, 8)$

Solution:

$y = x + 5$

Exercise:

Problem: $m = 2$, $(1, 4)$

Exercise:

Problem: $m = 8$, $(4, 0)$

Solution:

$y = 8x - 32$

Exercise:

Problem: $m = -3$, $(3, 0)$

Exercise:

Problem: $m = -1$, $(6, 0)$

Solution:

$y = -x + 6$

Exercise:

Problem: $m = -6$, $(0, 0)$

Exercise:

Problem: $m = -2$, $(0, 1)$

Solution:

$y = -2x + 1$

Exercise:

Problem: $(0, 0)$, $(3, 2)$

Exercise:

Problem: $(0, 0)$, $(5, 8)$

Solution:

$y = \frac{8}{5}x$

Exercise:

Problem: $(4, 1)$, $(6, 3)$

Exercise:

Problem: $(2, 5)$, $(1, 4)$

Solution:

$y = x + 3$

Exercise:

Problem: $(5, -3)$, $(6, 2)$

Exercise:

Problem: $(2, 3)$, $(5, 3)$

Solution:

$y = 3$ (horizontal line)

Exercise:

Problem: $(-1, 5)$, $(4, 5)$

Exercise:

Problem: $(4, 1)$, $(4, 2)$

Solution:

$x = 4$ (vertical line)

Exercise:

Problem: $(2, 7)$, $(2, 8)$

Exercise:

Problem: $(3, 3)$, $(5, 5)$

Solution:

$y = x$

Exercise:

Problem: $(0, 0)$, $(1, 1)$

Exercise:

Problem: $(-2, 4)$, $(3, -5)$

Solution:

$y = -\frac{9}{5}x + \frac{2}{5}$

Exercise:

Problem: $(1, 6)$, $(-1, -6)$

Exercise:

Problem: $(14, 12)$, $(-9, -11)$

Solution:

$y = x - 2$

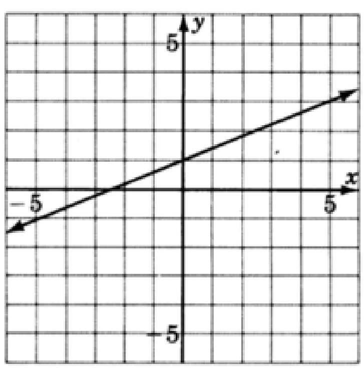
Exercise:

Problem: $(0, -4)$, $(5, 0)$

For the following problems, read only from the graph and determine the equation of the lines.

Exercise:

Problem:

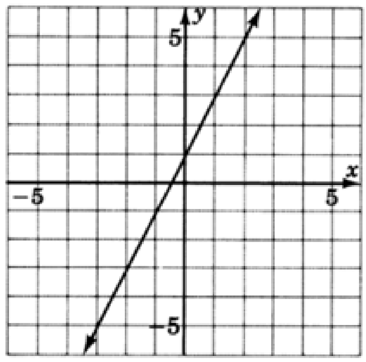


Solution:

$y = \frac{2}{5}x + 1$

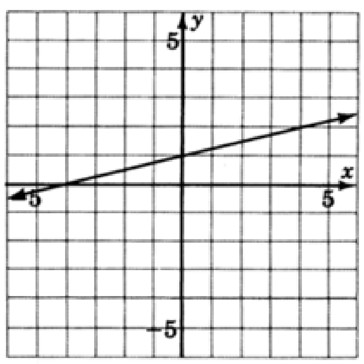
Exercise:

Problem:



Exercise:

Problem:

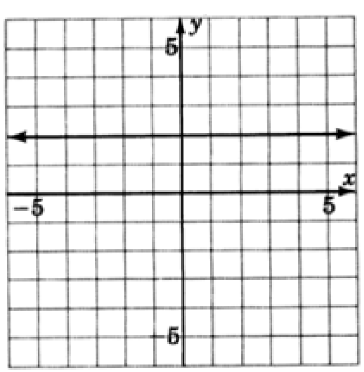


Solution:

$y = \frac{1}{4}x + 1$

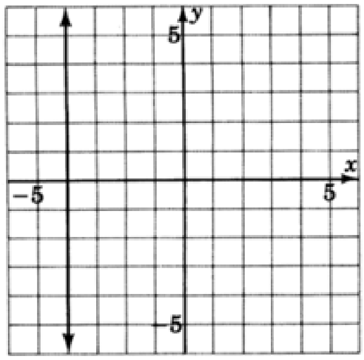
Exercise:

Problem:



Exercise:

Problem:

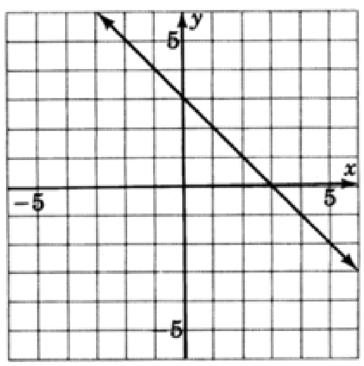


Solution:

$x = -4$

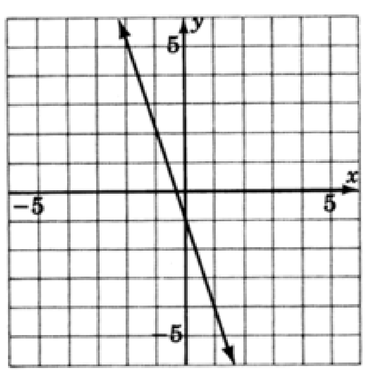
Exercise:

Problem:



Exercise:

Problem:



Solution:

$y = -3x - 1$

Exercises for Review

Exercise:

Problem: ([link]) Graph the equation $x - 3 = 0$.

Exercise:

Problem:

([link]) Supply the missing word. The point at which a line crosses the y-axisis called the .

Solution:

y-intercept

Exercise:

Problem:

([link]) Supply the missing word. The of a line is a measure of the steepness of the line.

Exercise:

Problem:

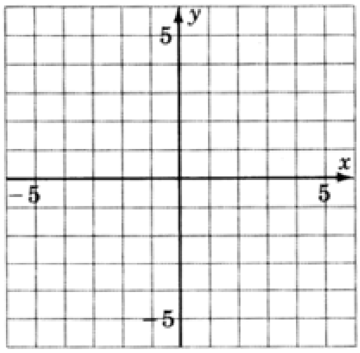
([link]) Find the slope of the line that passes through the points $(4, 0)$and $(-2, -6)$.

Solution:

$m = 1$

Exercise:

Problem: ([link]) Graph the equation $3y = 2x + 3$.


y
5
x
−5
5
−5

Graphing Slope Intercept Form
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. This module contains an overview of the chapter "Graphing Linear Equations and Inequalities in One and Two Variables".

Overview

- Using the Slope and Intercept to Graph a Line

Using the Slope and Intercept to Graph a Line

When a linear equation is given in the **general form**, $ax + by = c$, we observed that an efficient graphical approach was the intercept method. We let $x = 0$ and computed the corresponding value of y, then let $y = 0$ and computed the corresponding value of x.

When an equation is written in the **slope-intercept form**, $y = mx + b$, there are also efficient ways of constructing the graph. One way, but less efficient, is to choose two or three $x\text{-values}$ and compute to find the corresponding $y\text{-values}$. However, computations are tedious, time consuming, and can lead to errors. Another way, the method listed below, makes use of the slope and the $y\text{-intercept}$ for graphing the line. It is quick, simple, and involves no computations.

Graphing Method

1. Plot the $y\text{-intercept } (0,\ b)$.
2. Determine another point by using the $\text{slope}\ m$.
3. Draw a line through the two points.

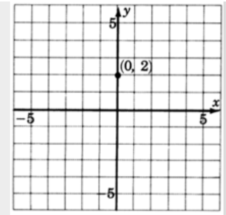
Recall that we defined the $\text{slope}\ m$ as the ratio $\frac{y_2 - y_1}{x_2 - x_1}$. The numerator $y_2 - y_1$ represents the number of units that y changes and the denominator $x_2 - x_1$ represents the number of units that x changes. Suppose $m = \frac{p}{q}$. Then p is the number of units that y changes and q is the number of units that x changes. Since these changes occur simultaneously, start with your pencil at the $y\text{-intercept}$, move p units in the appropriate vertical direction, and then move q units in the appropriate horizontal direction. Mark a point at this location.

Sample Set A

Graph the following lines.

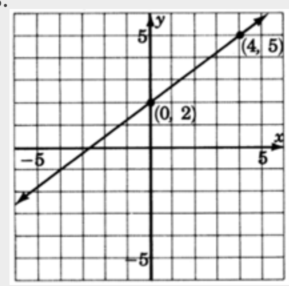
Example:
$y = \frac{3}{4}x + 2$

The $y\text{-intercept}$	is the point	$(0,\ 2)$. Thus the line crosses the	$y\text{-axis}$	2 units above the origin. Mark a point at $(0,\ 2)$.



The slope, m, is $\frac{3}{4}$. This means that if we start at any point on the line and move our pencil 3 units up and then 4 units to the right, we'll be back on the line. Start at a known point, the y-intercept $(0, 2)$. Move up 3 units, then move 4 units to the right. Mark a point at this location. (Note also that $\frac{3}{4} = \frac{-3}{-4}$. This means that if we start at any point on the line and move our pencil 3 units **down** and 4 units **left**, to the

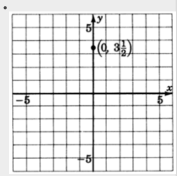
Draw a line through both points.



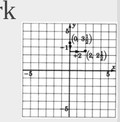
Example:

$y = -\frac{1}{2}x + \frac{7}{2}$

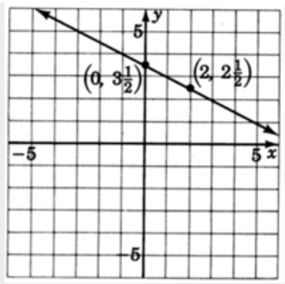
The y-intercept is the point $\left(0,\ \frac{7}{2}\right)$. Thus the line crosses the y-axis $\frac{7}{2}$ units above the origin. Mark a point at $\left(0,\ \frac{7}{2}\right)$, or $\left(0,\ 3\frac{1}{2}\right)$.



The slope, m, is $-\frac{1}{2}$. We can write $-\frac{1}{2}$ as $\frac{-1}{2}$. Thus, we start at a known point, the y-intercept $\left(0,\ 3\frac{1}{2}\right)$, move **down** one unit (because of the -1), then move right 2 units. Mark a point at this location.

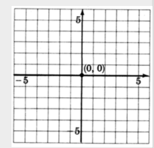


Draw a line through both points.

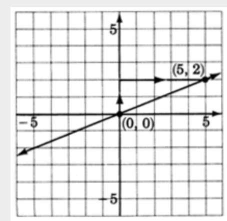


Example:
$y = \frac{2}{5}x$

We can put this equation into explicit slope-intercept by writing it as $y = \frac{2}{5}x + 0$. The y-intercept is the point $(0,\ 0)$, the origin. This line goes right through the origin.



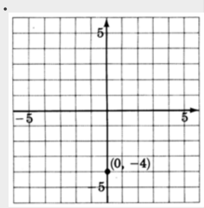
The slope, m, is $\frac{2}{5}$. Starting at the origin, we move up 2 units, then move to the right 5 units. Mark a point at this location.



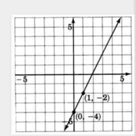
Draw a line through the two points.

Example:
$y = 2x - 4$

The y-intercept is the point $(0,\ -4)$. Thus the line crosses the y-axis 4 units below the origin. Mark a point at $(0,\ -4)$.



The slope, m, is 2. If we write the slope as a fraction, $2 = \frac{2}{1}$, we can read how to make the changes. Start at the known point $(0,\ -4)$, move up 2 units, then move right 1 unit. Mark a point at this location.



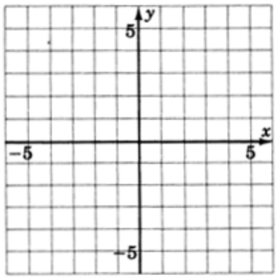
Draw a line through the two points.

Practice Set A

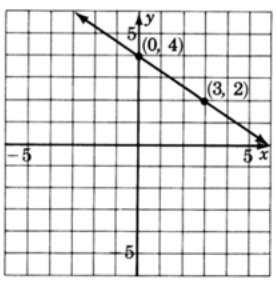
Use the y-intercept and the slope to graph each line.

Exercise:

Problem: $y = \frac{-2}{3}x + 4$

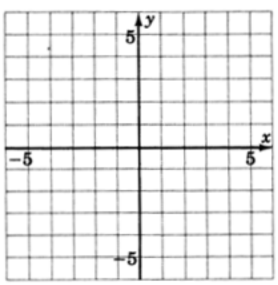


Solution:

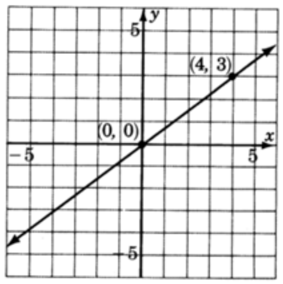


Exercise:

Problem: $y = \frac{3}{4}x$



Solution:



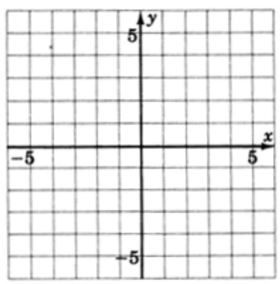
Excercises

For the following problems, graph the equations.

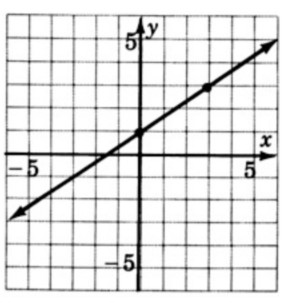
Exercise:

$$y = \frac{2}{3}x + 1$$

Problem:



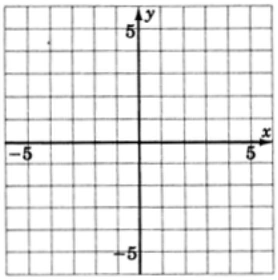
Solution:



Exercise:

$$y = \frac{1}{4}x - 2$$

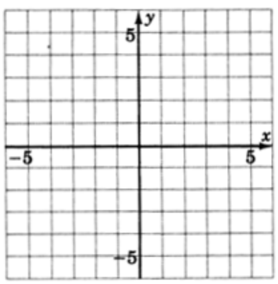
Problem:



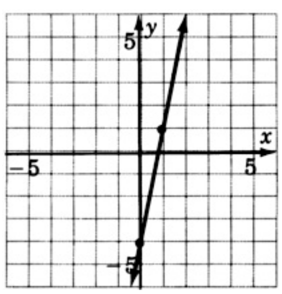
Exercise:

$$y = 5x - 4$$

Problem:



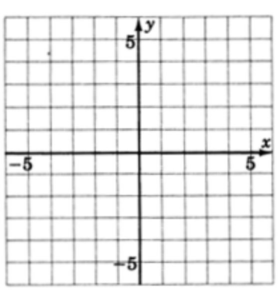
Solution:



Exercise:

$$y = -\frac{6}{5}x - 3$$

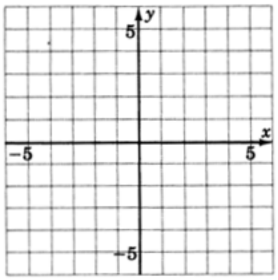
Problem:



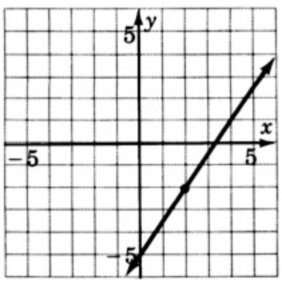
Exercise:

$$y = \frac{3}{2}x - 5$$

Problem:



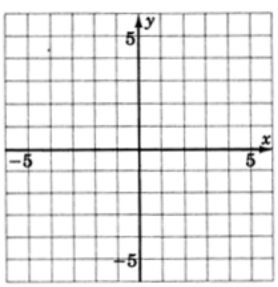
Solution:



Exercise:

$$y = \frac{1}{5}x + 2$$

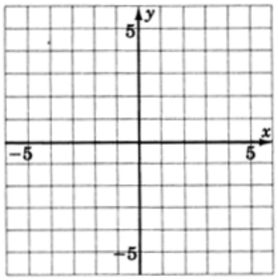
Problem:



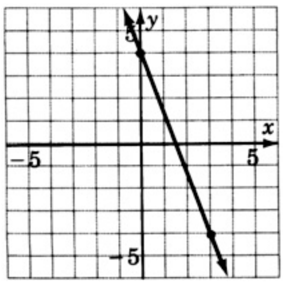
Exercise:

$$y = -\frac{8}{3}x + 4$$

Problem:



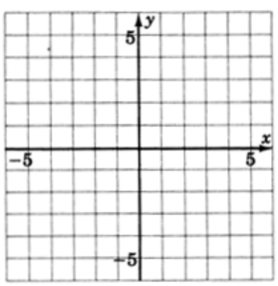
Solution:



Exercise:

$$y = -\frac{10}{3}x + 6$$

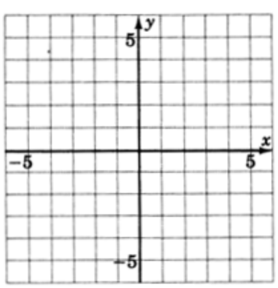
Problem:



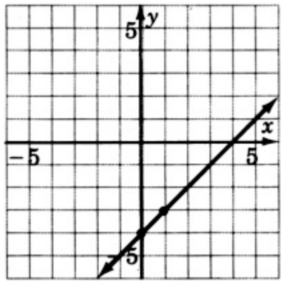
Exercise:

$$y = 1x - 4$$

Problem:



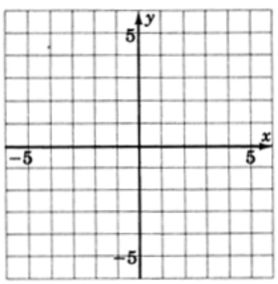
Solution:



Exercise:

$$y = -2x + 1$$

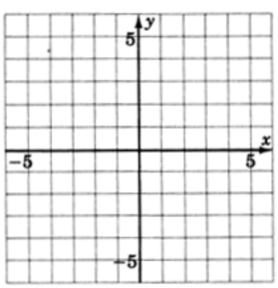
Problem:



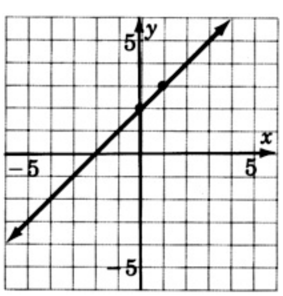
Exercise:

$$y = x + 2$$

Problem:



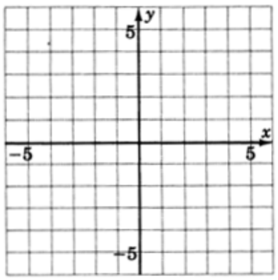
Solution:



Exercise:

$$y=\frac{3}{5}x$$

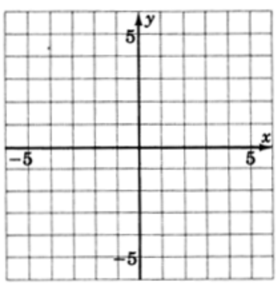
Problem:



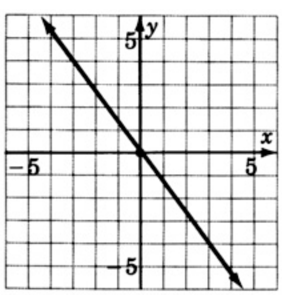
Exercise:

$$y=-\frac{4}{3}x$$

Problem:



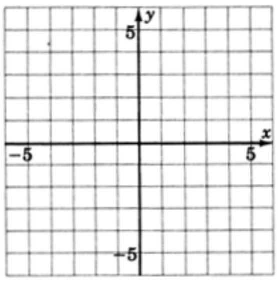
Solution:



Exercise:

$$y=x$$

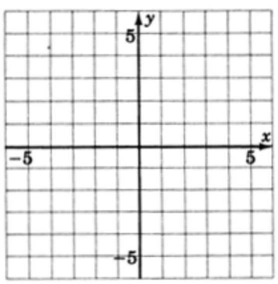
Problem:



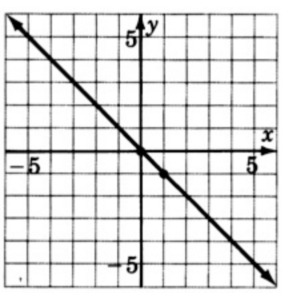
Exercise:

Problem:

$$y = -x$$



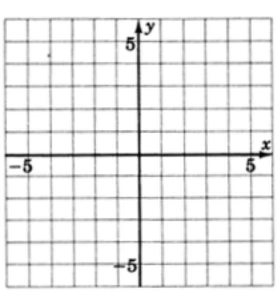
Solution:



Exercise:

Problem:

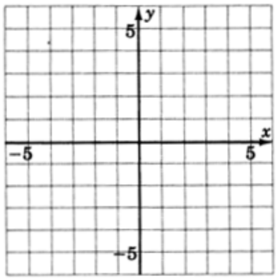
$$3y - 2x = -3$$



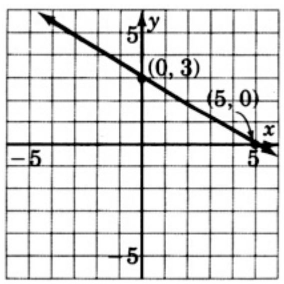
Exercise:

$$6x + 10y = 30$$

Problem:



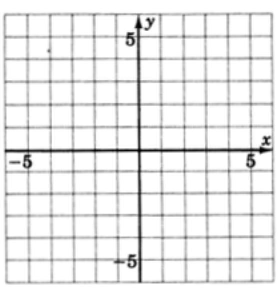
Solution:



Exercise:

$$x + y = 0$$

Problem:



Excersise for Review

Exercise:

Problem: ([link]) Solve the inequality $2 - 4x \geq x - 3$.

Solution:

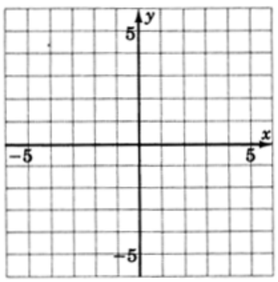
$x \leq 1$

Exercise:

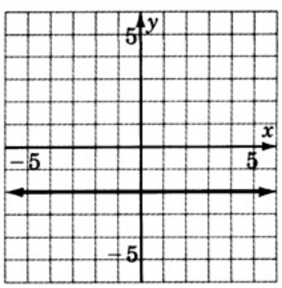
Problem: (**[link]**) Graph the inequality $y+3>1$.

Exercise:

Problem: (**[link]**) Graph the equation $y=-2$.



Solution:



Exercise:

Problem: (**[link]**) Determine the slope and y-intercept of the line $-4y-3x=16$.

Exercise:

Problem: (**[link]**) Find the slope of the line passing through the points $(-1,\,5)$ and $(2,\,3)$.

Solution:

$m=\frac{-2}{3}$

Graphing Slope Intercept Form of a Line
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: be more familiar with the general form of a line, be able to recognize the slope-intercept form of a line, be able to interpret the slope and intercept of a line, be able to use the slope formula to find the slope of a line.

Overview

- The General Form of a Line
- The Slope-Intercept Form of a Line
- Slope and Intercept
- The Formula for the Slope of a Line

The General Form of a Line

We have seen that the general form of a linear equation in two variables is $ax+by=c$ (Section [link]). When this equation is solved for y, the resulting form is called the slope-intercept form. Let's generate this new form.

$$\begin{array}{rcll}
ax+by & = & c & \text{Subtract } ax \text{ from both sides.} \\
by & = & -ax+c & \qquad \text{Divide } both \text{ sides by } b \\
\frac{by}{b} & = & \frac{-ax}{b}+\frac{c}{b} & \\
\frac{\cancel{b}y}{\cancel{b}} & = & \frac{-ax}{b}+\frac{c}{b} & \\
y & = & \frac{-ax}{b}+\frac{c}{b} & \\
y & = & \frac{-ax}{b}+\frac{c}{b} &
\end{array}$$

This equation is of the form $y=mx+b$ if we replace $\frac{-a}{b}$ with m and constant $\frac{c}{b}$ with b. (**Note:** The fact that we let $b=\frac{c}{b}$ is unfortunate and occurs beacuse of the letters we have chosen to use in the general form. The letter b occurs on both sides of the equal sign and may not represent the same value at all. This problem is one of the historical convention and, fortunately, does not occur very often.)

The following examples illustrate this procedure.

Example:
Solve $3x+2y=6$ for y.

$$\begin{aligned} 3x + 2y &= 6 && \text{Subtract } 3x \text{ from both sides.} \\ 2y &= -3x + 6 && \text{Divide both sides by 2.} \\ y &= -\frac{3}{2}x + 3 \end{aligned}$$

This equation is of the form $y = mx + b$. In this case, $m = -\frac{3}{2}$ and $b = 3$.

Example:
Solve $-15x + 5y = 20$ for y.

$$\begin{aligned} -15x + 5y &= 20 \\ 5y &= 15x + 20 \\ y &= 3x + 4 \end{aligned}$$

This equation is of the form $y = mx + b$. In this case, $m = 3$ and $b = 4$.

Example:
Solve $4x - y = 0$ for y.

$$\begin{aligned} 4x - y &= 0 \\ -y &= -4x \\ y &= 4x \end{aligned}$$

This equation is of the form $y = mx + b$. In this case, $m = 4$ and $b = 0$. Notice that we can write $y = 4x$ as $y = 4x + 0$.

The Slope-Intercept Form of a Line

The Slope-Intercept Form of a Line $y = mx + b$
A linear equation in two variables written in the form $y = mx + b$ is said to be in **slope-intercept form.**

Sample Set A

The following equations **are** in slope-intercept form:

Example:
$y = 6x - 7.\quad \text{In this case } m = 6 \text{ and } b = -7.$

Example:
$y = -2x + 9.\quad \text{In this case } m = -2 \text{ and } b = 9.$

Example:
$y = \frac{1}{5}x + 4.8\quad \text{In this case } m = \frac{1}{5} \text{ and } b = 4.8.$

Example:
$y = 7x$. In this case $m = 7$ and $b = 0$ since we can write $y = 7x$ as $y = 7x + 0$.

The following equations **are not** in slope-intercept form:

Example:
$2y = 4x - 1$. The coefficient of y is 2. To be in slope-intercept form, the coefficient of y must be 1.

Example:
$y + 4x = 5$. The equation is not solved for y. The x and y appear on the same side of the equal sign.

Example:
$y + 1 = 2x$. The equation is not solved for y.

Practice Set A

The following equation are in slope-intercept form. In each case, specify the slope and y-intercept.
Exercise:

Problem: $y = 2x + 7; \quad m = \qquad b =$

Solution:

$m = 2,\ b = 7$

Exercise:

Problem: $y = -4x + 2; \quad m = \qquad b =$

Solution:

$m = -4, b = 2$

Exercise:

Problem: $y = -5x - 1; \quad m = \qquad b =$

Solution:

$m = -5, b = -1$

Exercise:

Problem: $y = \frac{2}{3}x - 10; \quad m = \qquad\qquad b =$

Solution:

$m = \frac{2}{3}$, $b = \ -10$

Exercise:

Problem: $y = \frac{-5}{8}x + \frac{1}{2}; \quad m = \qquad\qquad b =$

Solution:

$m = \frac{-5}{8}$, $b = \frac{1}{2}$

Exercise:

Problem: $y = -3x; \quad m = \qquad\qquad b =$

Solution:

$m = -3$, $b = 0$

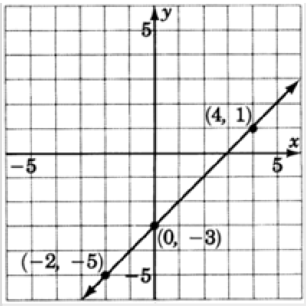
Slope and Intercept

When the equation of a line is written in slope-intercept form, two important properties of the line can be seen: the **slope** and the **intercept**. Let's look at these two properties by graphing several lines and observing them carefully.

Sample Set B

Example:
Graph the line $y = x - 3$.

x	y	$(x,\ y)$
0	-3	$(0,\ -3)$
4	1	$(4,\ 1)$
-2	-5	$(-2,\ -5)$



Looking carefully at this line, answer the following two questions.
Exercise:

Problem: At what number does this line cross the y-axis? Do you see this number in the equation?

Solution:

The line crosses the y-axis at -3.

Exercise:

Problem:

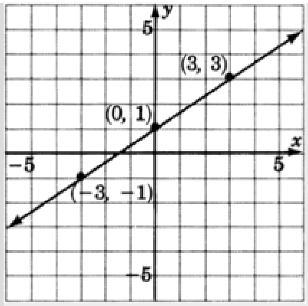
Place your pencil at any point on the line. Move your pencil exactly **one** unit horizontally to the right. Now, how many units straight up or down must you move your pencil to get back on the line? Do you see this number in the equation?

Solution:

After moving horizontally one unit to the right, we must move exactly one vertical unit up. This number is the coefficient of x.

Example:
Graph the line $y = \frac{2}{3}x + 1$.

x	y	$(x,\ y)$
0	1	$(0,\ 1)$
3	3	$(3,\ 3)$
-3	-1	$(-3,\ -1)$



Looking carefully at this line, answer the following two questions.
Exercise:

Problem: At what number does this line cross the y-axis? Do you see this number in the equation?

Solution:

The line crosses the y-axis at $+1$.

Exercise:

Problem:

Place your pencil at any point on the line. Move your pencil exactly **one** unit horizontally to the right. Now, how many units straight up or down must you move your pencil to get back on the line? Do you see this number in the equation?

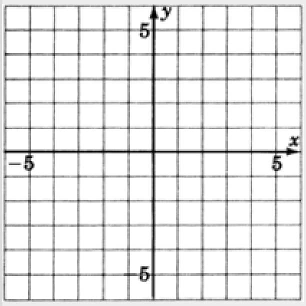
Solution:

After moving horizontally one unit to the right, we must move exactly $\frac{2}{3}$ unit upward. This number is the coefficient of x.

Practice Set B

Example:
Graph the line $y = -3x + 4$.

x	y	(x, y)
0		
3		
2		



Looking carefully at this line, answer the following two questions.

Exercise:

Problem: At what number does the line cross the y-axis? Do you see this number in the equation?

Solution:

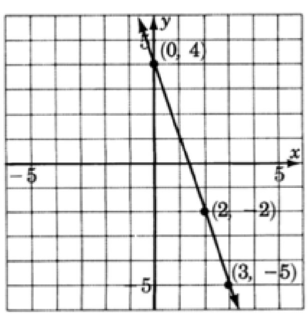
The line crosses the y-axis at $+4$. After moving horizontally 1 unit to the right, we must move exactly 3 units downward.

Exercise:

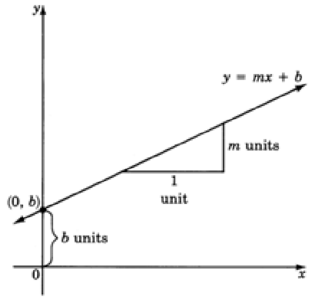
Problem:

Place your pencil at any point on the line. Move your pencil exactly **one** unit horizontally to the right. Now, how many units straight up or down must you move your pencil to get back on the line? Do you see this number in the equation?

Solution:



In the graphs constructed in Sample Set B and Practice Set B, each equation had the form $y = mx + b$. We can answer the same questions by using this form of the equation (shown in the diagram).



y-Intercept

Exercise:

Problem: At what number does the line cross the y-axis? Do you see this number in the equation?

Solution:

In each case, the line crosses the y-axis at the constant b. The number b is the number at which the line crosses the y-axis, and it is called the $y\text{-intercept}$. The ordered pair corresponding to the $y\text{-intercept is }(0,\ b)$.

Exercise:

Problem:

Place your pencil at any point on the line. Move your pencil exactly **one** unit horizontally to the right. Now, how many units straight up or down must you move your pencil to get back on the line? Do you see this number in the equation?

Solution:

To get back on the line, we must move our pencil exactly m vertical units.

Slope

The number m is the coefficient of the variable x. The number m is called the **slope** of the line and it is the number of units that y changes when x is increased by 1 unit. Thus, if x changes by 1 unit, y changes by m units.

Since the equation $y=mx+b$ contains both the slope of the line and the $y\text{-intercept}$, we call the form $y=mx+b$ the **slope-intercept** form.

The Slope-Intercept Form of the Equation of a Line

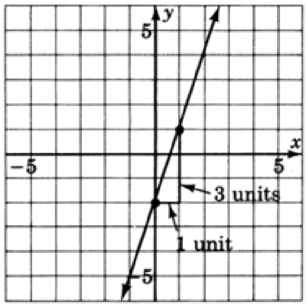
The slope-intercept form of a straight line is

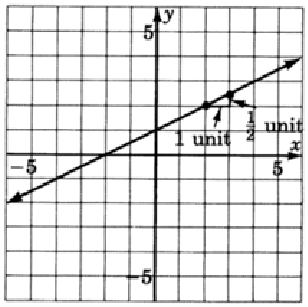
$y=mx+b$

The slope of the line is m, and the $y\text{-intercept}$ is the point $(0\ ,\ b)$.

The Slope is a Measure of the Steepness of a Line

The word **slope** is really quite appropriate. It gives us a measure of the steepness of the line. Consider two lines, one with slope $\frac{1}{2}$ and the other with slope 3. The line with slope 3 is steeper than is the line with slope $\frac{1}{2}$. Imagine your pencil being placed at any point on the lines. We make a 1-unit increase in the x-value by moving the pencil **one** unit to the right. To get back to one line we need only move vertically $\frac{1}{2}$ unit, whereas to get back onto the other line we need to move vertically 3 units.





Sample Set C

Find the slope and the y-intercept of the following lines.

Example:
$y = 2x + 7.$

The line is in the slope-intercept form $y = mx + b$. The slope is m, the coefficient of x. Therefore, $m = 2$. The y-intercept is the point $(0,\ b)$. Since $b = 7$, the y-intercept is $(0,\ 7)$.
Slope : 2

y-intercept : $(0,\ 7)$

Example:
$y = -4x + 1.$

The line is in slope-intercept form $y = mx + b$. The slope is m, the coefficient of x. So, $m = -4$. The y-intercept is the point $(0,\ b)$. Since $b = 1$, the y-intercept is $(0,\ 1\ )$.
Slope : -4

y-intercept : $(0,\ 1)$

Example:
$3x + 2y = 5.$

The equation is written in general form. We can put the equation in slope-intercept form by solving for y.

$$\begin{aligned} 3x + 2y &= 5 \\ 2y &= -3x + 5 \\ y &= -\frac{3}{2}x + \frac{5}{2} \end{aligned}$$

Now the equation is in slope-intercept form.
Slope: $-\frac{3}{2}$
y-intercept: $\left(0,\ \frac{5}{2}\right)$

Practice Set C

Exercise:

Problem: Find the slope and y-intercept of the line $2x + 5y = 15$.

Solution:

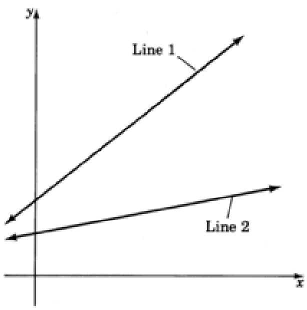
Solving for y we get $y = \frac{-2}{5}x + 3$. Now, $m = \frac{-2}{5}$ and $b = 3$.

The Formula for the Slope of a Line

We have observed that the slope is a measure of the steepness of a line. We wish to develop a formula for measuring this steepness.

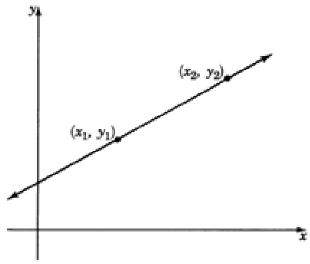
It seems reasonable to develop a slope formula that produces the following results:

Steepness of line 1 $>$ steepness of line 2.

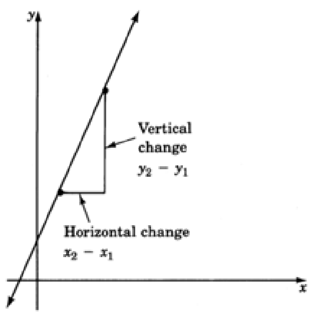


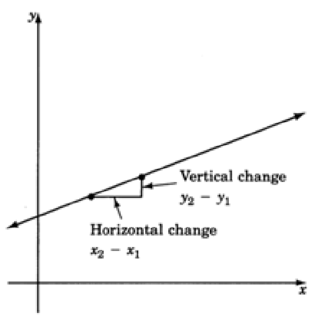
Consider a line on which we select any two points. We'll denote these points with the ordered pairs $(x_{1,} y_1)$ and $(x_{2,} y_2)$. The subscripts help us to identify the points.

$(x_{1,} y_1)$ is the first point. Subscript 1 indicates the first point.
$(x_{2,} y_2)$ is the second point. Subscript 2 indicates the second point.



The difference in x values $(x_2 - x_1)$ gives us the horizontal change, and the difference in y values $(y_2 - y_1)$ gives us the vertical change. If the line is very steep, then when going from the first point to the second point, we would expect a large vertical change compared to the horizontal change. If the line is not very steep, then when going from the first point to the second point, we would expect a small vertical change compared to the horizontal change.





We are comparing changes. We see that we are comparing

The vertical change	to	the horizontal change
The change in y	to	the change in x
$y_2 - y_1$	to	$x_2 - x_1$

This is a comparison and is therefore a **ratio**. Ratios can be expressed as fractions. Thus, a measure of the steepness of a line can be expressed as a ratio.

The slope of a line is defined as the ratio

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

Mathematically, we can write these changes as

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding the Slope of a Line
The slope of a nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is found by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

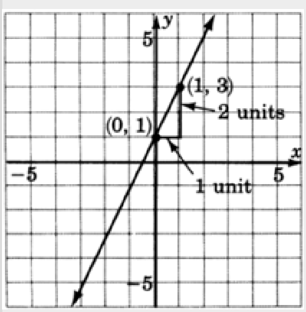
Sample Set D

For the two given points, find the slope of the line that passes through them.

Example:
$(0, 1)$ and $(1, 3)$.

Looking left to right on the line we can choose (x_1, y_1) to be $(0, 1)$, and (x_2, y_2) to be $(1, 3)$.Then,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{1 - 0} = \frac{2}{1} = 2$$



This line has slope 2. It appears fairly steep. When the slope is written in fraction form, $2 = \frac{2}{1}$, we can see, by recalling the slope formula, that as x changes 1 unit to the right (because of the $+1$) y changes 2 units upward (because of the $+2$).

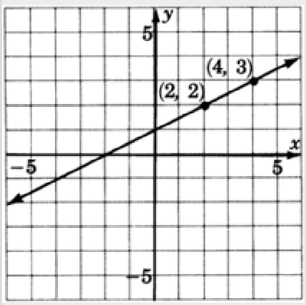
$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{2}{1}$$

Notice that as we look left to right, the line rises.

Example:
$(2, 2)$ and $(4, 3)$.

Looking left to right on the line we can choose $(x_{1,} y_1)$ to be $(2, 2)$ and $(x_{2,} y_2)$ to be $(4, 3)$. Then,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{4 - 2} = \frac{1}{2}$$



This line has slope $\frac{1}{2}$. Thus, as x changes 2 units to the right (because of the $+2$), y changes 1 unit upward (because of the $+1$).

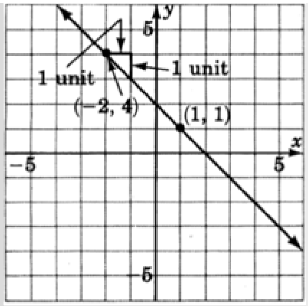
$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{1}{2}$$

Notice that in examples 1 and 2, both lines have positive slopes, $+2$ and $+\frac{1}{2}$, and both lines **rise** as we look left to right.

Example:
$(-2,\ 4)$ and $(1,\ 1)$.

Looking left to right on the line we can choose $(x_{1,} y_1)$ to be $(-2,\ 4)$ and $(x_{2,} y_2)$ to be $(1,\ 1)$. Then,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{1 - (-2)} = \frac{-3}{1 + 2} = \frac{-3}{3} = -1$$

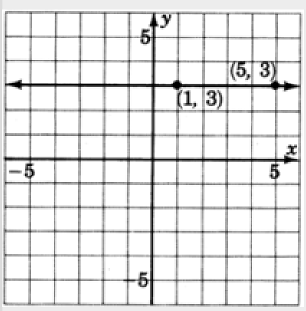


This line has slope -1.

When the slope is written in fraction form, $m = -1 = \frac{-1}{+1}$, we can see that as x changes 1 unit to the right (because of the $+1$), y changes 1 unit downward (because of the -1).
Notice also that this line has a negative slope and declines as we look left to right.

Example:
$(1, 3)$ and $(5, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{5 - 1} = \frac{0}{4} = 0$$

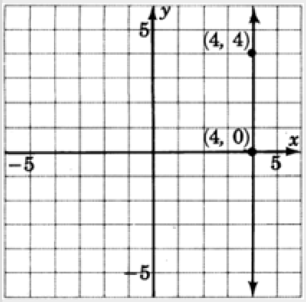


This line has 0 slope. This means it has **no** rise and, therefore, is a horizontal line. This does not mean that the line has no slope, however.

Example:
$(4, 4)$ and $(4, 0)$.

This problem shows why the slope formula is valid only for nonvertical lines.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{4 - 4} = \frac{-4}{0}$$



Since division by 0 is undefined, we say that vertical lines have undefined slope. Since there is no real number to represent the slope of this line, we sometimes say that vertical lines have **undefined slope**, or **no slope**.

Practice Set D

Exercise:

Problem:

Find the slope of the line passing through $(2, 1)$ and $(6, 3)$. Graph this line on the graph of problem 2 below.

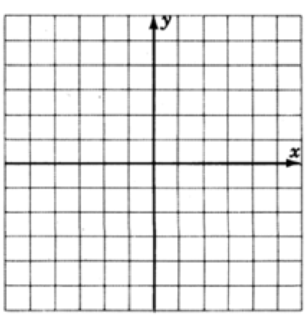
Solution:

$m = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$.

Exercise:

Find the slope of the line passing through $(3, 4)$ and $(5, 5)$. Graph this line.

Problem:



Solution:

The line has slope $\frac{1}{2}$.

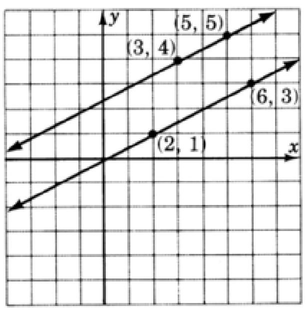
Exercise:

Problem:

Compare the lines of the following problems. Do the lines appear to cross? What is it called when lines do not meet (parallel or intersecting)? Compare their slopes. Make a statement about the condition of these lines and their slopes.

Solution:

The lines appear to be parallel. Parallel lines have the same slope, and lines that have the same slope are parallel.



Before trying some problems, let's summarize what we have observed.

Exercise:

Problem:

The equation $y = mx + b$ is called the slope-intercept form of the equation of a line. The number m is the slope of the line and the point $(0, b)$ is the $y\text{-intercept}$.

Exercise:

Problem:

The slope, m, of a line is defined as the steepness of the line, and it is the number of units that y changes when x changes 1 unit.

Exercise:

Problem:

The formula for finding the slope of a line through any two given points (x_1, y_1) and $(x_2, y_2)$is

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Exercise:

Problem: The fraction $\frac{y_2 - y_1}{x_2 - x_1}$ represents the $\frac{\text{Change in } y}{\text{Change in } x}$.

Exercise:

Problem:

As we look at a graph from left to right, lines with positive slope rise and lines with negative slope decline.

Exercise:

Problem: Parallel lines have the same slope.

Exercise:

Problem: Horizontal lines have 0 slope.

Exercise:

Problem: Vertical lines have undefined slope (or no slope).

Exercises

For the following problems, determine the slope and y-intercept of the lines.

Exercise:

Problem: $y = 3x + 4$

Solution:

$\text{slope} = 3$; $y\text{-intercept} = (0, 4)$

Exercise:

Problem: $y = 2x + 9$

Exercise:

Problem: $y = 9x + 1$

Solution:

$\text{slope} = 9$; $y\text{-intercept} = (0, 1)$

Exercise:

Problem: $y = 7x + 10$

Exercise:

Problem: $y = -4x + 5$

Solution:

$\text{slope} = -4$; $y\text{-intercept} = (0, 5)$

Exercise:

Problem: $y = -2x + 8$

Exercise:

Problem: $y = -6x - 1$

Solution:

$\text{slope} = -6$; $y\text{-intercept} = (0, -1)$

Exercise:

Problem: $y = -x - 6$

Exercise:

Problem: $y = -x + 2$

Solution:

$\text{slope} = -1$; $y\text{-intercept} = (0, 2)$

Exercise:

Problem: $2y = 4x + 8$

Exercise:

Problem: $4y = 16x + 20$

Solution:

$\text{slope} = 4$; $y\text{-intercept} = (0, 5)$

Exercise:

Problem: $-5y = 15x + 55$

Exercise:

Problem: $-3y = 12x - 27$

Solution:

$\text{slope} = -4$; $y\text{-intercept} = (0, 9)$

Exercise:

Problem: $y = \frac{3}{5}x - 8$

Exercise:

Problem: $y=\frac{2}{7}x-12$

Solution:

$\text{slope}=\frac{2}{7};\ y\text{-intercept}=(0,-12)$

Exercise:

Problem: $y=\frac{-1}{8}x+\frac{2}{3}$

Exercise:

Problem: $y=\frac{-4}{5}x-\frac{4}{7}$

Solution:

$\text{slope}=-\frac{4}{5};\ y\text{-intercept}=\left(0,-\frac{4}{7}\right)$

Exercise:

Problem: $-3y=5x+8$

Exercise:

Problem: $-10y=-12x+1$

Solution:

$\text{slope}=\frac{6}{5};\ y\text{-intercept}=\left(0,-\frac{1}{10}\right)$

Exercise:

Problem: $-y=x+1$

Exercise:

Problem: $-y=-x+3$

Solution:

$\text{slope}=1;\ y\text{-intercept}=(0,-3)$

Exercise:

Problem: $3x-y=7$

Exercise:

Problem: $5x+3y=6$

Solution:

slope $= -\frac{5}{3}$; y-intercept $= (0, 2)$

Exercise:

Problem: $-6x - 7y = -12$

Exercise:

Problem: $-x + 4y = -1$

Solution:

slope $= \frac{1}{4}$; y-intercept $= \left(0, -\frac{1}{4}\right)$

For the following problems, find the slope of the line through the pairs of points.

Exercise:

Problem: $(1,\ 6),\ (4, 9)$

Exercise:

Problem: $(1,\ 3),\ (4, 7)$

Solution:

$m = \frac{4}{3}$

Exercise:

Problem: $(3,\ 5),\ (4, 7)$

Exercise:

Problem: $(6,\ 1),\ (2, 8)$

Solution:

$m = -\frac{7}{4}$

Exercise:

Problem: $(0,\ 5),\ (2, -6)$

Exercise:

Problem: $(-2,\ 1),\ (0, 5)$

Solution:

$m = 2$

Exercise:

Problem: $(3,\ -9),\ (5,1)$

Exercise:

Problem: $(4,\ -6),\ (-2,1)$

Solution:

$m = -\frac{7}{6}$

Exercise:

Problem: $(-5,\ 4),\ (-1,0)$

Exercise:

Problem: $(-3,\ 2),\ (-4,6)$

Solution:

$m = -4$

Exercise:

Problem: $(9,\ 12),\ (6,0)$

Exercise:

Problem: $(0,\ 0),\ (6,6)$

Solution:

$m = 1$

Exercise:

Problem: $(-2,\ -6),\ (-4,-1)$

Exercise:

Problem: $(-1,\ -7),\ (-2,-9)$

Solution:

$m = 2$

Exercise:

Problem: $(-6,\ -6),\ (-5,-4)$

Exercise:

Problem: $(-1,\ 0),\ (-2,-2)$

Solution:

$m = 2$

Exercise:

Problem: $(-4,\ -2),\ (0,0)$

Exercise:

Problem: $(2,\ 3),\ (10,3)$

Solution:

$m = 0\ \ (\text{ horizontal line } y = 3)$

Exercise:

Problem: $(4,\ -2),\ (4,7)$

Exercise:

Problem: $(8,\ -1),\ (8,3)$

Solution:

$\text{No slope } (\text{vertical line at } x = 8)$

Exercise:

Problem: $(4,\ 2),\ (6,2)$

Exercise:

Problem: $(5,\ -6),\ (9,-6)$

Solution:

$m = 0\ \ (\text{ horizontal line at } y = -6)$

Exercise:

Problem: Do lines with a positive slope rise or decline as we look left to right?

Exercise:

Problem: Do lines with a negative slope rise or decline as we look left to right?

Solution:

decline

Exercise:

Problem: Make a statement about the slopes of parallel lines.

Calculator Problems

For the following problems, determine the slope and y-intercept of the lines. Round to two decimal places.

Exercise:

Problem: $3.8x + 12.1y = 4.26$

Solution:

$\text{slope} = -0.31$
$y - \text{intercept} = (0, 0.35)$

Exercise:

Problem: $8.09x + 5.57y = -1.42$

Exercise:

Problem: $10.813x - 17.0y = -45.99$

Solution:

$\text{slope} = 0.64$
$y - \text{intercept} = (0, 2.71)$

Exercise:

Problem: $-6.003x - 92.388y = 0.008$

For the following problems, find the slope of the line through the pairs of points. Round to two decimal places.

Exercise:

Problem: $(5.56,\ 9.37),\ (2.16,\ 4.90)$

Solution:

$m = 1.31$

Exercise:

Problem: $(33.1,\ 8.9),\ (42.7,\ -1.06)$

Exercise:

Problem: $(155.89,\ 227.61),\ (157.04, 227.61)$

Solution:

$m = 0\ \ (\text{ horizontal line at } y = 227.61)$

Exercise:

Problem: $(0.00426,\ \ -0.00404),\ (-0.00191,\ \ -0.00404)$

Exercise:

Problem: $(88.81,\ \ -23.19),\ (88.81,\ \ -26.87)$

Solution:

$\text{No slope}\ \ (\text{vertical line } x = 88.81)$

Exercise:

Problem: $(-0.0000567,\ \ -0.0000567),\ (-0.00765,\ 0.00764)$

Exercises for Review

Exercise:

Problem: ([link]) Simplify $\left(x^2y^3w^4\right)^0$.

Solution:

$1 \text{ if } xyw \neq 0$

Exercise:

Problem: ([link]) Solve the equation $3x - 4(2 - x) - 3(x - 2) + 4 = 0$.

Exercise:

Problem:

([link]) When four times a number is divided by five, and that result is decreased by eight, the result is zero. What is the original number?

Solution:

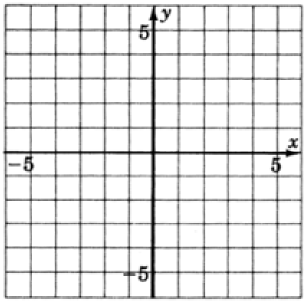
10

Exercise:

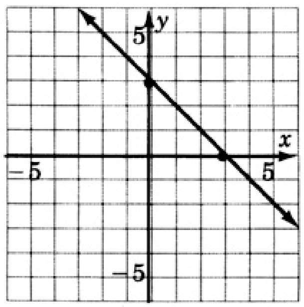
Problem: ([link]) Solve $-3y + 10 = x + 2\ \text{ if } x = -4$.

Exercise:

Problem: ([link]) Graph the linear equation $x + y = 3$.



Solution:



Graphing One Variable Inequalities
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: understand the concept of a graph and the relationship between axes, coordinate systems, and dimension, be able to construct one-dimensional graphs.

Overview

- Graphs
- Axes, Coordinate Systems, and Dimension
- Graphing in One Dimension

Graphs

We have, thus far in our study of algebra, developed and used several methods for obtaining solutions to linear equations in both one and two variables. Quite often it is helpful to obtain a picture of the solutions to an equation. These pictures are called **graphs** and they can reveal information that may not be evident from the equation alone.

The Graph of an Equation

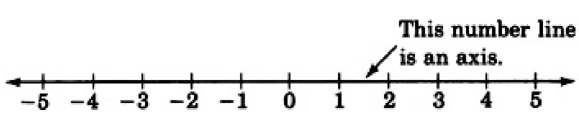
The geometric representation (picture) of the solutions to an equation is called the **graph** of the equation.

Axes, Coordinate Systems, and Dimension

Axis

The basic structure of the graph is the **axis**. It is with respect to the axis that all solutions to an equation are located. The most fundamental type of axis is the **number line**.

The Number Line is an Axis



We have the following general rules regarding axes.

Number of Variables and Number of Axes

- An equation in one variable requires one axis.
- An equation in two variables requires two axes.
- An equation in three variables requires three axes.
- ... An equation in n variables requires n axes.

We shall always draw an axis as a straight line, and if more than one axis is required, we shall draw them so they are all mutually perpendicular (the lines forming the axes will be at 90° angles to one another).

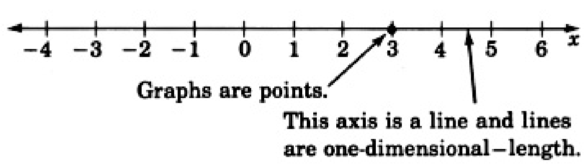
Coordinate System
A system of axes constructed for graphing an equation is called a **coordinate system**.

The Phrase, Graphing an Equation
The phrase **graphing an equation** is used frequently and should be interpreted as meaning geometrically locating the solutions to an equation.

Relating the Number of Variables and the Number of Axes
We will not start actually graphing equations until Section [link], but in the following examples we will **relate** the number of variables in an equation to the number of axes in the coordinate system.

- **1. One-Dimensional Graphs**

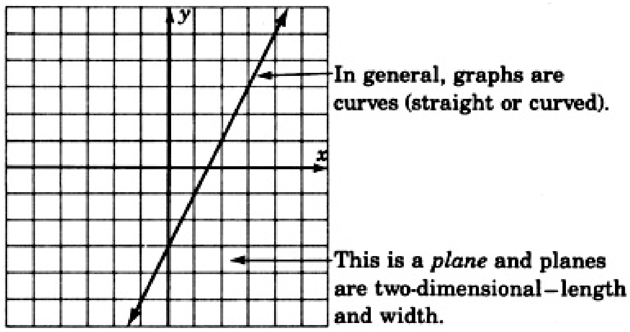
  If we wish to graph the equation $5x + 2 = 17$, we would need to construct a coordinate system consisting of a single axis (a single number line) since the equation consists of only one variable. We label the axis with the variable that appears in the equation.

  We might interpret an equation in one variable as giving information in one-dimensional space. Since we live in three-dimensional space, one-dimensional space might be hard to imagine. Objects in one-dimensional space would have only length, no width or depth.
- **2. Two-Dimensional Graphs**

  To graph an equation in two variables such as $y = 2x – 3$, we would need to construct a coordinate system consisting of two mutually perpendicular number lines **(axes).** We call the intersection of the two axes the **origin** and label it with a 0. The two axes are

simply number lines; one drawn horizontally, one drawn vertically.

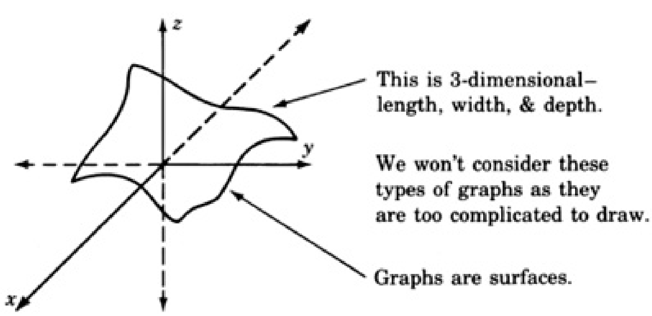


Recall that an equation in two variables requires a solution to be a pair of numbers. The solutions can be written as ordered pairs (x, y). Since the equation $y = 2x - 3$ involves the variables x and y, we label one axis x and the other axis y. In mathematics it is customary to label the horizontal axis with the independent variable and the vertical axis with the dependent variable.

We might interpret equations in two variables as giving information in two-dimensional space. Objects in two-dimensional space would have length and width, but no depth.

- **3. Three-Dimensional Graphs**

An equation in three variables, such as $3x^2 - 4y^2 + 5z = 0$, requires three mutually perpendicular axes, one for each variable. We would construct the following coordinate system and graph.



We might interpret equations in three variables as giving information about three-dimensional space.

- **4. Four-Dimensional Graphs**

To graph an equation in four variables, such as $3x - 2y + 8x - 5w = -7$, would require four mutually perpendicular number lines. These graphs are left to the imagination.

We might interpret equations in four variables as giving information in four-dimensional space. Four-dimensional objects would have length, width, depth, and some other dimension.

Black Holes
These other spaces are hard for us to imagine, but the existence of “black holes” makes the possibility of other universes of one-, two-, four-, or n-dimensions not entirely unlikely. Although it may be difficult for us “3-D” people to travel around in another dimensional space, at least we could be pretty sure that our mathematics would still work (since it is not restricted to only three dimensions)!

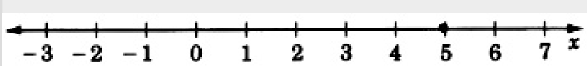
Graphing in One Dimension

Graphing a linear equation in one variable involves solving the equation, then locating the solution on the axis (number line), and marking a point at this location. We have observed that graphs may reveal information that may not be evident from the original equation. The graphs of linear equations in one variable do not yield much, if any, information, but they serve as a foundation to graphs of higher dimension (graphs of two variables and three variables).

Sample Set A

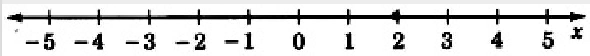
Example:
Graph the equation $3x – 5 = 10$.
Solve the equation for x and construct an axis. Since there is only one variable, we need only one axis. Label the axis x.

$$\begin{array}{lcl} 3x – 5 & = & 10 \\ 3x & = & 15 \\ x & = & 5 \end{array}$$



Example:
Graph the equation $3x + 4 + 7x – 1 + 8 = 31$.
Solving the equation we get,

$$\begin{aligned} 10x + 11 &= 31 \\ 10x &= 20 \\ x &= 2 \end{aligned}$$



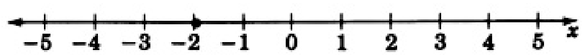
Practice Set A

Exercise:

Problem: Graph the equation $4x + 1 = -7$.

Solution:

$x = -2$

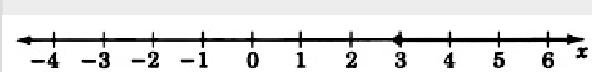


Sample Set B

Example:
Graph the linear inequality $4x \geq 12$.
We proceed by solving the inequality.

$$\begin{aligned} 4x &\geq 12 \quad \text{Divide each side by 4.} \\ x &\geq 3 \end{aligned}$$

As we know, any value greater than or equal to 3 will satisfy the original inequality. Hence we have infinitely many solutions and, thus, infinitely many points to mark off on our graph.

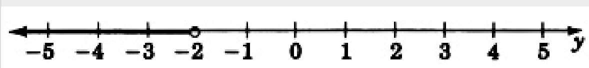


The **closed circle** at 3 means that 3 is included as a solution. All the points beginning at 3 and in the direction of the arrow are solutions.

Example:
Graph the linear inequality $-2y-1 > 3$.
We first solve the inequality.

$$\begin{array}{lll} -2y-1 & > & 3 \\ -2y & > & 4 \\ y & < & -2 \quad \text{The inequality symbol reversed direction because we divided by } -2. \end{array}$$

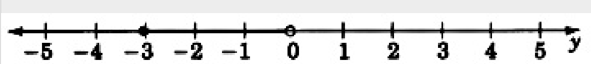
Thus, all numbers strictly less than -2 will satisfy the inequality and are thus solutions. Since -2 itself is **not** to be included as a solution, we draw an **open circle** at -2. The solutions are to the left of -2 so we draw an arrow pointing to the left of -2 to denote the region of solutions.



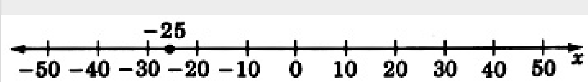
Example:
Graph the inequality $-2 \le y+1 < 1$.
We recognize this inequality as a **compound inequality** and solve it by subtracting 1 from all three parts.

$$\begin{array}{c} -2 \le y+1 < 1 \\ -3 \le y < 0 \end{array}$$

Thus, the solution is all numbers between -3 and 0, more precisely, all numbers greater than or equal to -3 but strictly less than 0.



Example:
Graph the linear equation $5x = -125$.
The solution is $x = -25$. Scaling the axis by units of 5 rather than 1, we obtain



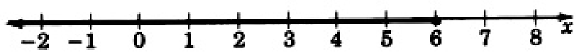
Practice Set B

Exercise:

Problem: Graph the inequality $3x \leq 18$.

Solution:

$x \leq 6$

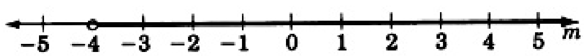


Exercise:

Problem: Graph the inequality $-3m + 1 < 13$.

Solution:

$m > -4$

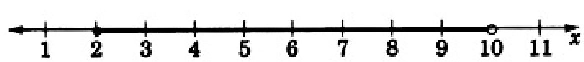


Exercise:

Problem: Graph the inequality $-3 \leq x - 5 < 5$.

Solution:

$2 \leq x < 10$

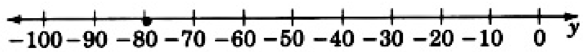


Exercise:

Problem: Graph the linear equation $-6y = 480$.

Solution:

$y = -80$



Exercises

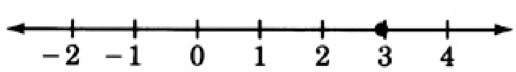
For problems 1 - 25, graph the linear equations and inequalities.

Exercise:

Problem: $4x + 7 = 19$

Solution:

$x = 3$



Exercise:

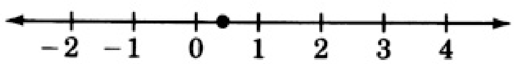
Problem: $8x - 1 = 7$

Exercise:

Problem: $2x + 3 = 4$

Solution:

$x = \frac{1}{2}$



Exercise:

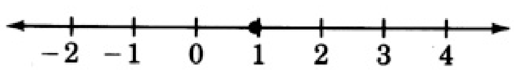
Problem: $x + 3 = 15$

Exercise:

Problem: $6y + 3 = y + 8$

Solution:

$y = 1$



Exercise:

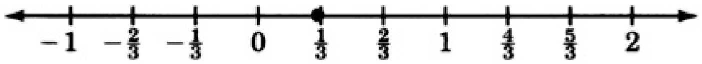
Problem: $2x = 0$

Exercise:

Problem: $4 + 1 - 4 = 3z$

Solution:

$z = \frac{1}{3}$



Exercise:

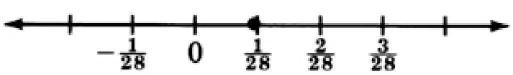
Problem: $x + \frac{1}{2} = \frac{4}{3}$

Exercise:

Problem: $7r = \frac{1}{4}$

Solution:

$r = \frac{1}{28}$



Exercise:

Problem: $2x - 6 = \frac{2}{5}$

Exercise:

Problem: $x + 7 \leq 12$

Solution:

$x \leq 5$

Exercise:

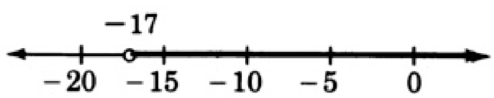
Problem: $y-5<3$

Exercise:

Problem: $x+19>2$

Solution:

$x>-17$



Exercise:

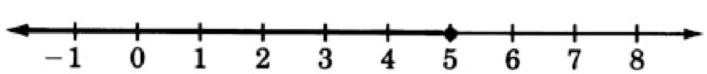
Problem: $z+5>11$

Exercise:

Problem: $3m-7\leq 8$

Solution:

$m\leq 5$



Exercise:

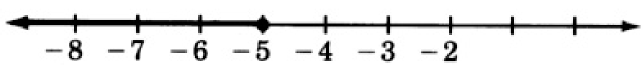
Problem: $-5t \geq 10$

Exercise:

Problem: $-8x - 6 \geq 34$

Solution:

$x \leq -5$



Exercise:

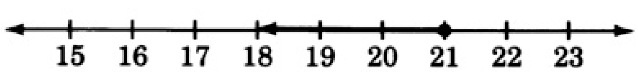
Problem: $\frac{x}{4} < 2$

Exercise:

Problem: $\frac{y}{7} \leq 3$

Solution:

$y \leq 21$



Exercise:

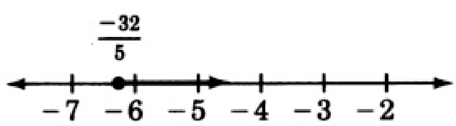
Problem: $\frac{2y}{9} \geq 4$

Exercise:

Problem: $\frac{-5y}{8} \leq 4$

Solution:

$y \geq -\frac{32}{5}$



Exercise:

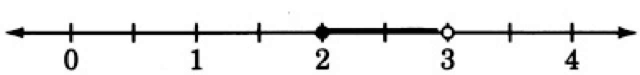
Problem: $\frac{-6a}{7} < -4$

Exercise:

Problem: $-1 \leq x - 3 < 0$

Solution:

$2 \leq x < 3$



Exercise:

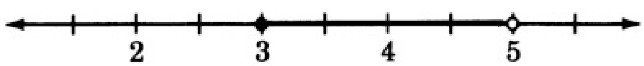
Problem: $6 \leq x + 4 \leq 7$

Exercise:

Problem: $-12 < -2x - 2 \leq -8$

Solution:

$3 \leq x < 5$



Exercises for Review

Exercise:

Problem: (**[link]**) Simplify $\left(3x^8y^2\right)^3$.

Exercise:

Problem:

(**[link]**) List, if any should appear, the common factors in the expression $10x^4 - 15x^2 + 5x^6$.

Solution:

$5x^2$

Exercise:

Problem: (**[link]**) Solve the inequality $-4(x + 3) < -3x + 1$.

Exercise:

Problem: (**[link]**) Solve the equation $y = -5x + 8$ if $x = -2$.

Solution:

$(-2, 18)$

Exercise:

Problem: **([link])** Solve the equation $2y = 5(3x + 7)$ if $x = -1$.

Graphing Two Variable Inequalities
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: be able to locate solutions to linear inequalities in two variables using graphical techniques.

Overview

- Location of Solutions
- Method of Graphing

Location of Solutions

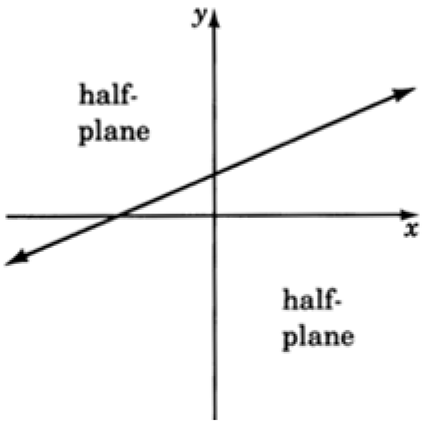
In our study of linear equations in two variables, we observed that **all** the solutions to the equation, and only the solutions to the equation, were located on the graph of the equation. We now wish to determine the location of the solutions to linear inequalities in two variables. Linear inequalities in two variables are inequalities of the forms:

$$\begin{array}{ll} ax+by\le c & ax+by\ge c \\ ax+by<c & ax+by>c \end{array}$$

Half-Planes

A straight line drawn through the plane divides the plane into two **half-planes**.

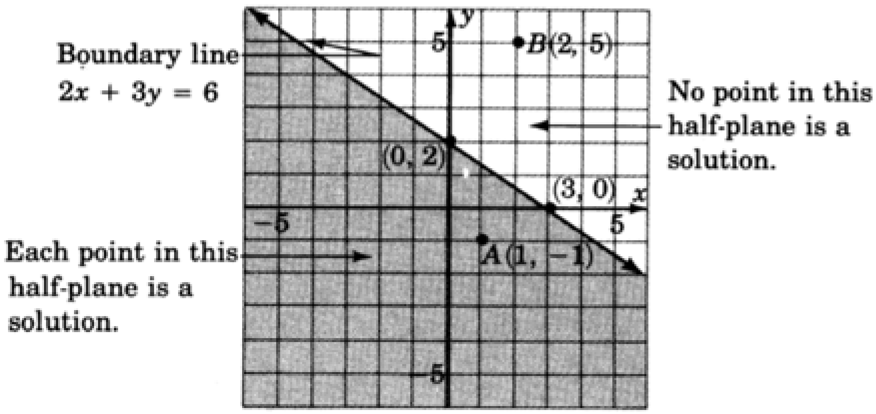
Boundary Line
The straight line is called the **boundary line**.



Solution to an Inequality in Two Variables
Recall that when working with linear equations in two variables, we observed that ordered pairs that produced true statements when substituted into an equation were called solutions to that equation. We can make a similar statement for inequalities in two variables. We say that an inequality in two variables has a solution when a pair of values has been found such that when these values are substituted into the inequality a true statement results.

The Location of Solutions in the Plane
As with equations, solutions to linear inequalities have particular locations in the plane. All solutions to a linear inequality in two variables are located in one and only in one entire half-plane. For example, consider the inequality

$$2x + 3y \leq 6$$



All the solutions to the inequality $2x + 3y \leq 6$ lie in the shaded half-plane.

Example:
Point $A(1,\ -1)$is a solution since
$2x + 3y \leq 6$

$2(1) + 3(-1) \leq 6?$

$2 - 3 \leq 6?$

$-1 \leq 6. \quad \text{True}$

Example:
Point $B(2,\ 5)$is not a solution since
$2x + 3y \leq 6$

$2(2) + 3(5) \leq 6?$

$4 + 15 \leq 6?$

$19 \leq 6. \qquad \text{False}$

Method of Graphing

The method of graphing linear inequalities in two variables is as follows:

1. Graph the boundary line (consider the inequality as an equation, that is, replace the inequality sign with an equal sign).

    a. If the inequality is $\leq$ or $\geq$, draw the boundary line **solid**. This means that points on the line are solutions and are part of the graph.
    b. If the inequality is $<$ or $>$, draw the boundary line **dotted**. This means that points on the line are **not** solutions and are **not** part of the graph.

2. Determine which half-plane to shade by choosing a test point.

    a. If, when substituted, the test point yields a true statement, shade the half-plane containing it.
    b. If, when substituted, the test point yields a false statement, shade the half-plane on the opposite side of the boundary line.

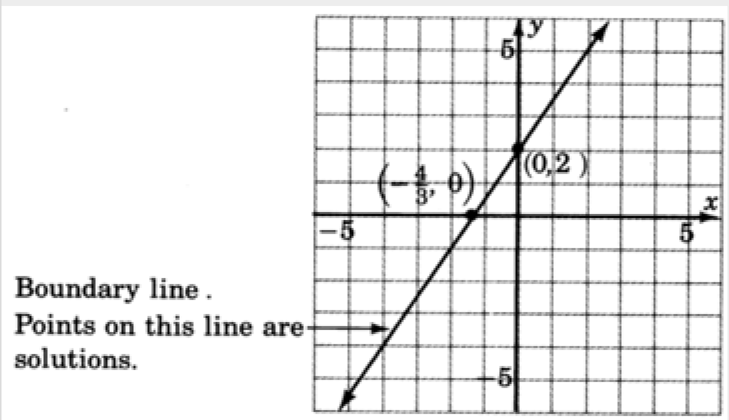
Sample Set A

Example:
Graph $3x - 2y \geq -4$.
1. Graph the boundary line. The inequality is $\geq$ so we'll draw the line **solid**. Consider the inequality as an equation.
Equation:

$$3x - 2y = -4$$

x	y	(x, y)

0	2	$(0, 2)$
$\frac{-4}{3}$	0	$\left(\frac{-4}{3}, 0\right)$

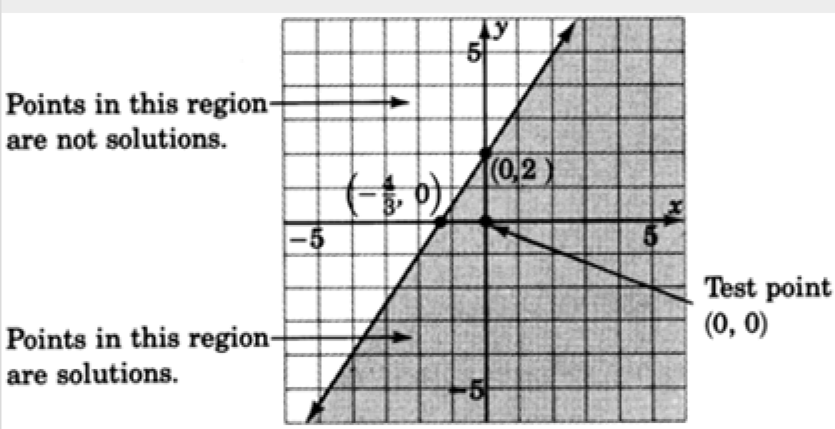


2. Choose a test point. The easiest one is $(0, 0)$. Substitute $(0, 0)$ into the original inequality.

Equation:

$$\begin{aligned} &3x - 2y \geq -4 \\ &3(0) - 2(0) \geq -4? \\ &0 - 0 \geq -4? \\ &0 \geq -4. \qquad \text{True} \end{aligned}$$

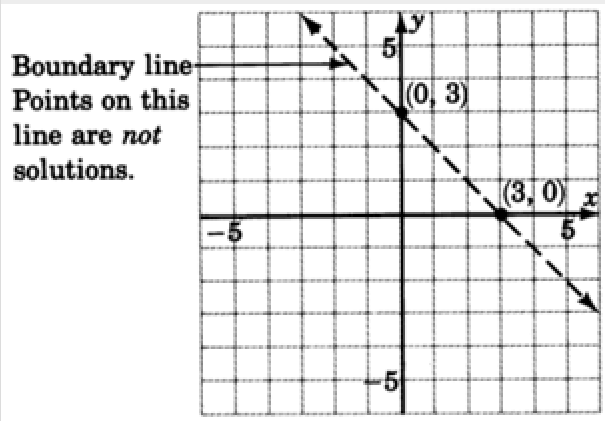
Shade the half-plane containing $(0, 0)$.



Example:

Graph $x + y - 3 < 0$.

1. Graph the boundary line: $x + y - 3 = 0$. The inequality is $<$ so we'll draw the line **dotted**.

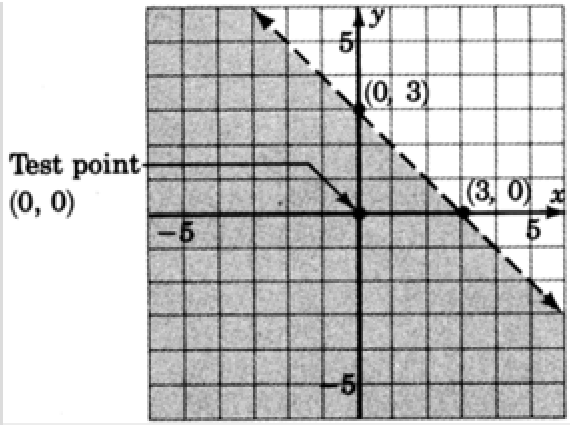


2. Choose a test point, say $(0,\ 0)$.

Equation:

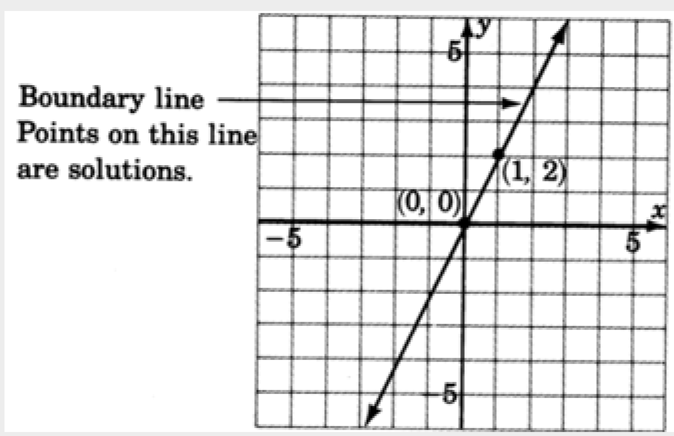
$$\begin{aligned} & x + y - 3 < 0 \\ & 0 + 0 - 3 < 0? \\ & -3 < 0. \qquad \text{True} \end{aligned}$$

Shade the half-plane containing $(0,\ 0)$.



Example:
Graph $y \leq 2x$.

1. Graph the boundary line $y = 2x$. The inequality is $\leq$, so we'll draw the line **solid**.



2. Choose a test point, say $(0,\ 0)$.

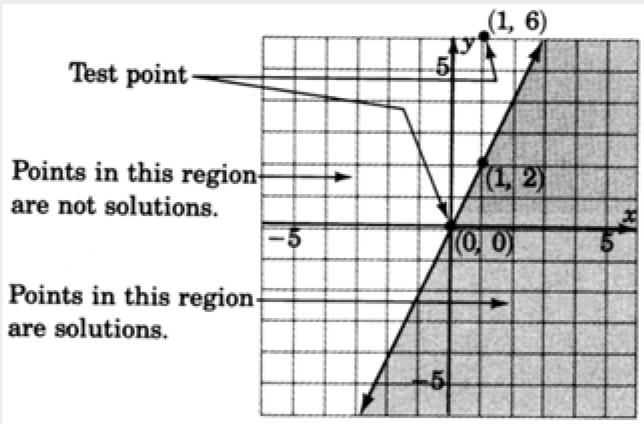
$y \leq 2x$
$0 \leq 2\,(0)?$
$0 \leq 0. \quad \text{True}$
Shade the half-plane containing $(0,\ 0)$. We can't! $(0,\ 0)$ is right on the line! Pick another test point, say $(1,\ 6)$.

$y \leq 2x$

$6 \leq 2\,(1)?$

$6 \leq 2. \quad \text{False}$

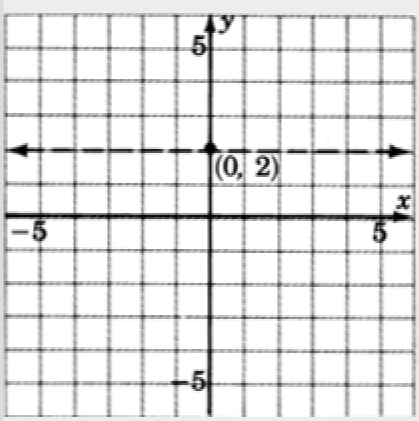
Shade the half-plane on the opposite side of the boundary line.



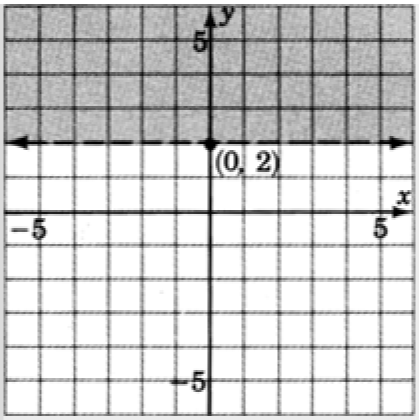
Example:

Graph $y > 2$.

1. Graph the boundary line $y = 2$. The inequality is $>$ so we'll draw the line **dotted**.



2. We don't really need a test point. Where is $y > 2$?**Above** the line $y = 2$! Any point above the line clearly has a y-coordinate greater than 2.



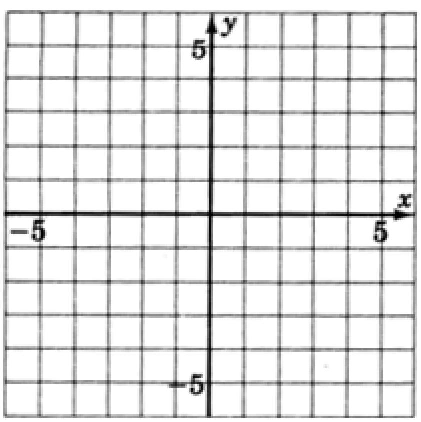
Practice Set A

Solve the following inequalities by graphing.

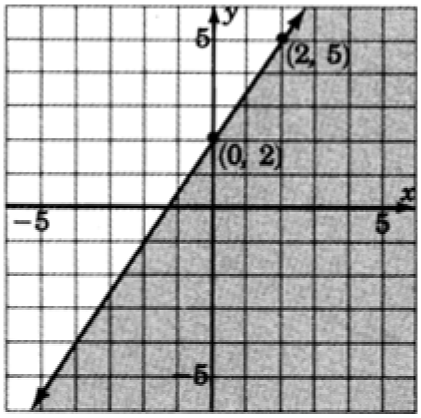
Exercise:

$$-3x + 2y \leq 4$$

Problem:



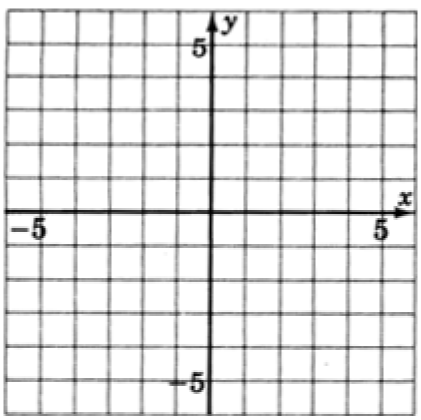
Solution:



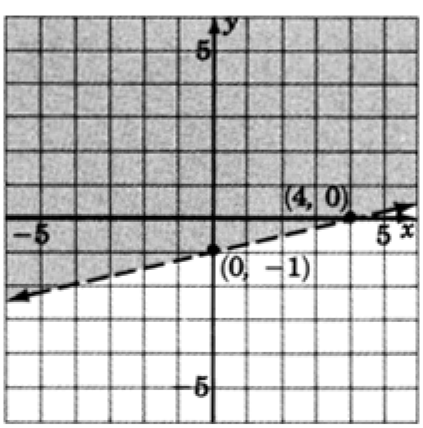
Exercise:

$$x - 4y < 4$$

Problem:



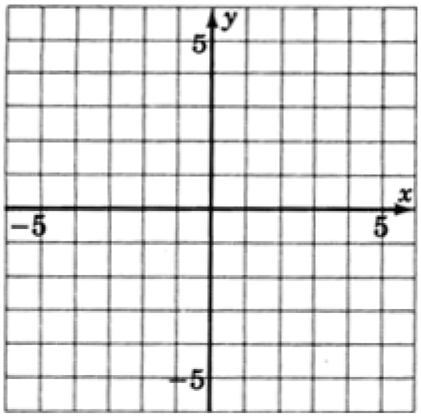
Solution:



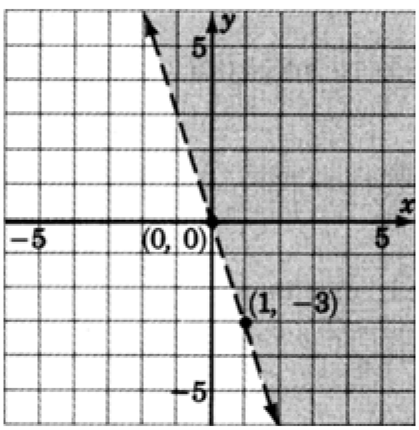
Exercise:

$$3x + y > 0$$

Problem:



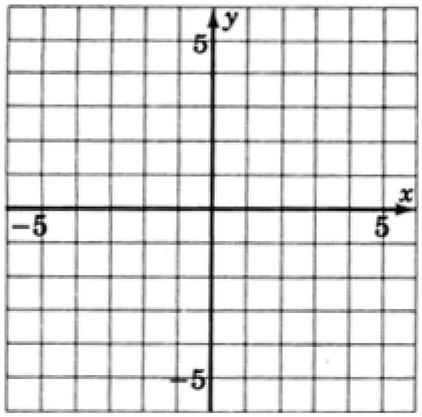
Solution:



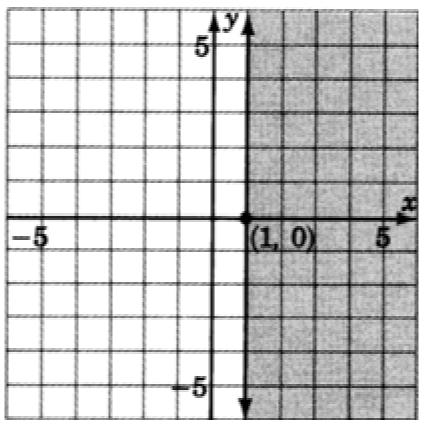
Exercise:

$$x \geq 1$$

Problem:



Solution:

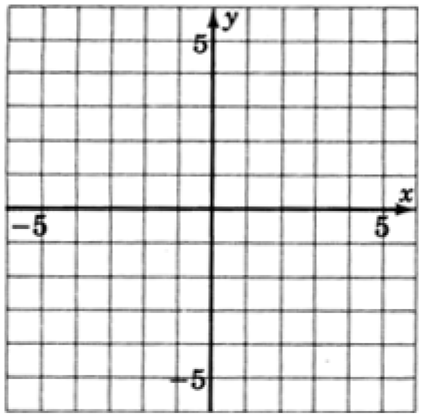


Exercises

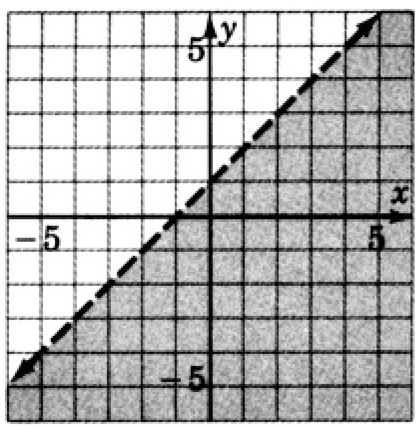
Solve the inequalities by graphing.
Exercise:

Problem:

$$y < x + 1$$



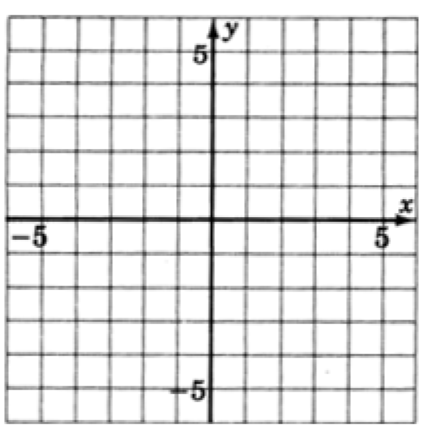
Solution:



Exercise:

$$x + y \leq 1$$

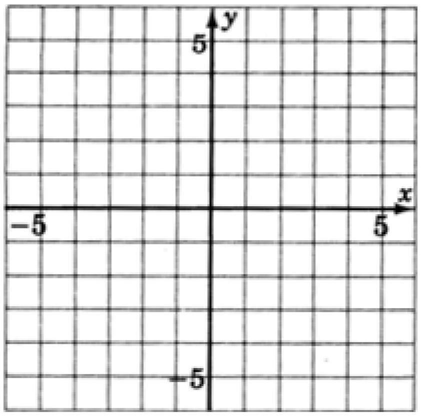
Problem:



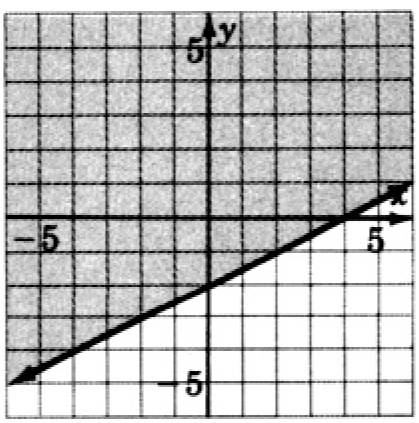
Exercise:

$$-x + 2y + 4 \geq 0$$

Problem:



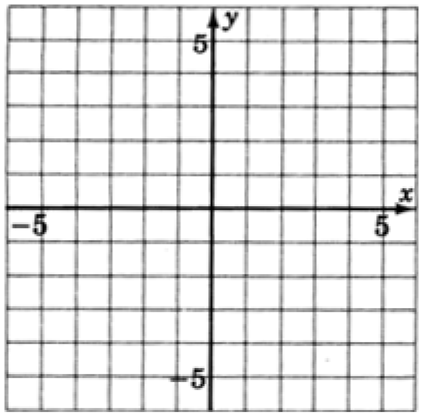
Solution:



Exercise:

$$-x + 5y - 10 < 0$$

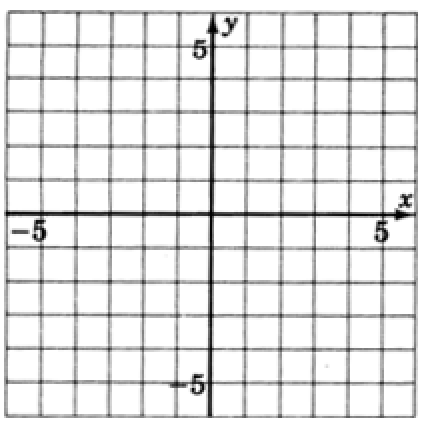
Problem:



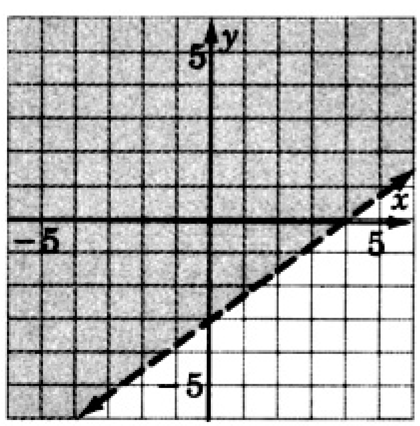
Exercise:

Problem:

$$-3x + 4y > -12$$



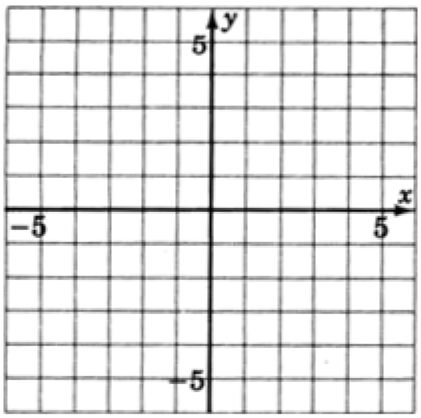
Solution:



Exercise:

$$2x + 5y - 15 \geq 0$$

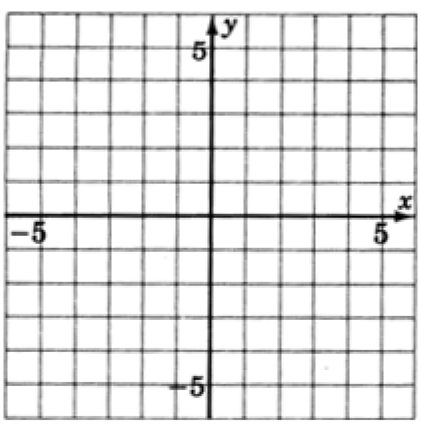
Problem:



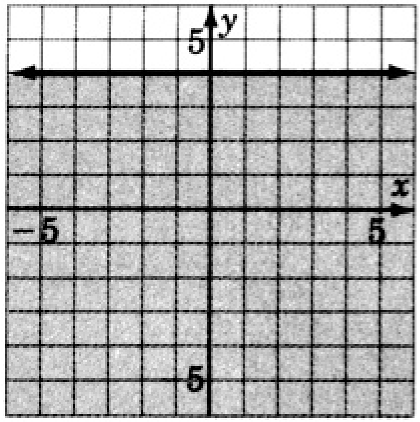
Exercise:

$$y \leq 4$$

Problem:

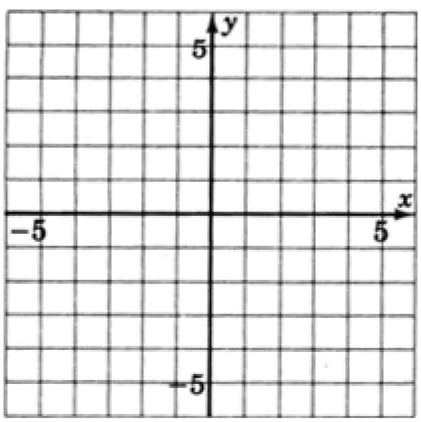


Solution:



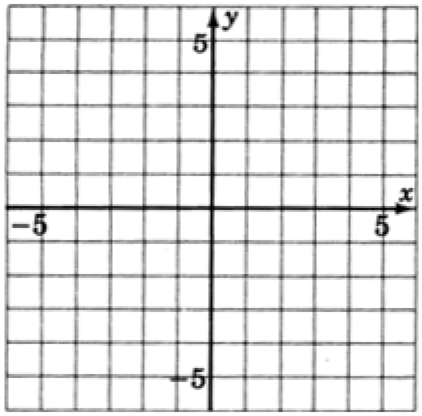
Exercise:

Problem: $x \geq 2$

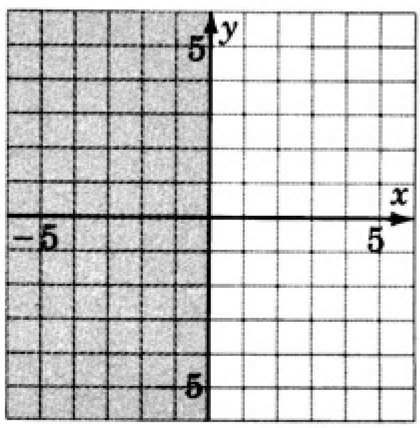


Exercise:

Problem: $x \leq 0$



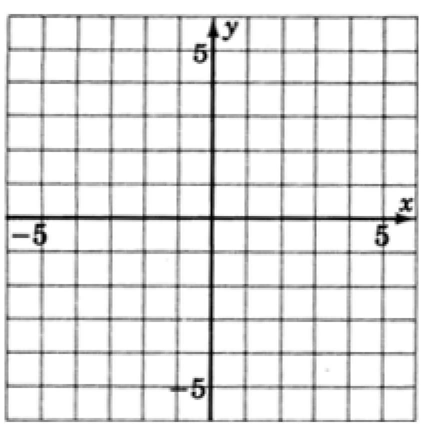
Solution:



Exercise:

$$x - y < 0$$

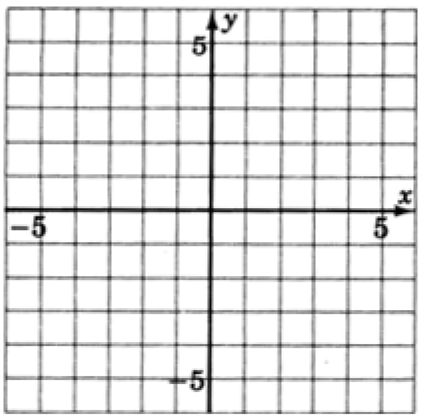
Problem:



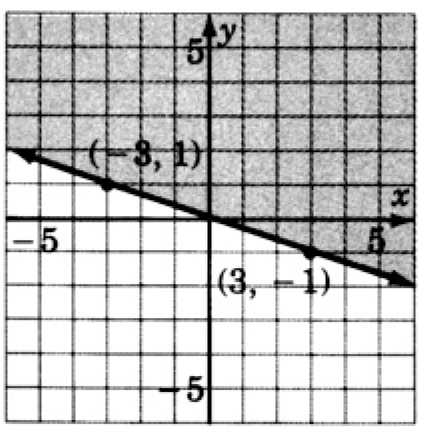
Exercise:

$$x+3y\geq 0$$

Problem:



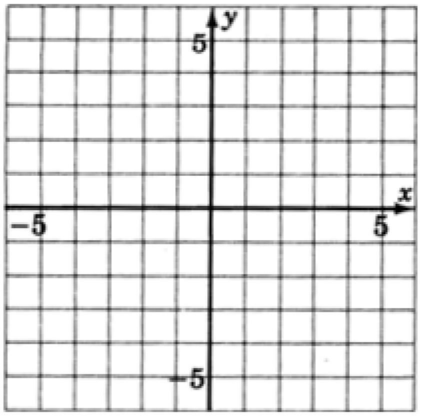
Solution:



Exercise:

$$-2x+4y>0$$

Problem:

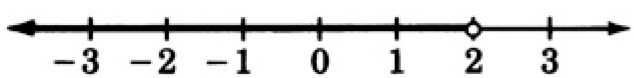


Exercises for Review

Exercise:

Problem: (**[link]**) Graph the inequality $-3x+5\geq -1$.

Solution:



Exercise:

Problem:

(**[link]**) Supply the missing word. The geometric representation (picture) of the solutions to an equation is called the of the equation.

Exercise:

Problem: (**[link]**) Supply the denominator:$m=\frac{y_2-y_1}{?}$.

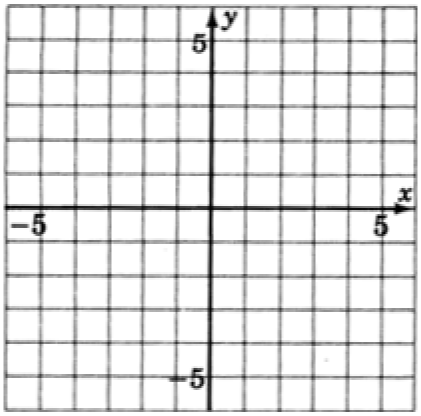
Solution:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Exercise:

Problem:

([link]) Graph the equation $y = -3x + 2$.



Exercise:

Problem:

([link]) Write the equation of the line that has slope 4 and passes through the point $(-1,\ 2)$.

Solution:

$y = 4x + 6$

Graphing Linear Inequalities
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: be able to relate solutions to a linear equation to lines, know the general form of a linear equation, be able to construct the graph of a line using the intercept method, be able to distinguish, by their equations, slanted, horizontal, and vertical lines.

Overview

- Solutions and Lines
- General form of a Linear Equation
- The Intercept Method of Graphing
- Graphing Using any Two or More Points
- Slanted, Horizontal, and Vertical Lines

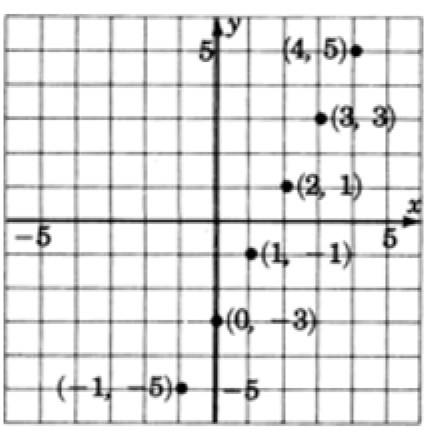
Solutions and Lines

We know that solutions to linear equations in two variables can be expressed as ordered pairs. Hence, the solutions can be represented by point in the plane. We also know that the phrase “graph the equation” means to locate the solution to the given equation in the plane. Consider the equation

$y - 2x = -3$. We'll graph six solutions (ordered pairs) to this equation on the coordinates system below. We'll find the solutions by choosing x-values (from -1 to $+4$), substituting them into the equation $y - 2x = -3$, and then solving to obtain the corresponding y-values. We can keep track of the ordered pairs by using a table.

$$y - 2x = -3$$

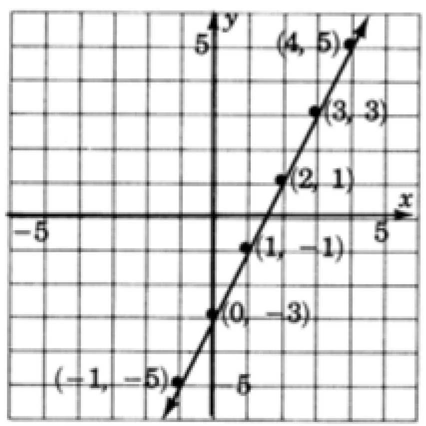
If $x =$	**Then $y =$**	**Ordered Pairs**
-1	-5	$(-1, -5)$
0	-3	$(0, -3)$
1	-1	$(1, -1)$
2	1	$(2, 1)$
3	3	$(3, 3)$
4	5	$(4, 5)$



We have plotted only six solutions to the equation $y - 2x = -3$. There are, as we know, infinitely many solutions. By observing the six points we have plotted, we can speculate as to the location of all the other points. The six points we plotted seem to lie on a straight line. This would lead us to believe that all the other points (solutions) also lie on that same line. Indeed, this is true. In fact, this is precisely why first-degree equations are called **linear** equations.

Linear Equations Produce Straight Lines

$$\begin{array}{c}\text{Line}\\ \updownarrow\\ \text{Linear}\end{array}$$



General Form of a Linear Equation

General Form of a Linear Equation in Two Variables

There is a standard form in which linear equations in two variables are written. Suppose that a, b, and c are any real numbers and that a and b cannot both be zero at the same time. Then, the linear equation in two variables

$$ax + by = c$$

is said to be in **general form**.

We must stipulate that a and b cannot both equal zero at the same time, for if they were we would have

$0x + 0y = c$

$0 = c$

This statement is true only if $c = 0$. If c were to be any other number, we would get a false statement.

Now, we have the following:

The graphing of all ordered pairs that solve a linear equation in two variables produces a straight line.

This implies,

The graph of a linear equation in two variables is a straight line.

From these statements we can conclude,

If an ordered pair is a solution to a linear equations in two variables, then it lies on the graph of the equation.

Also,

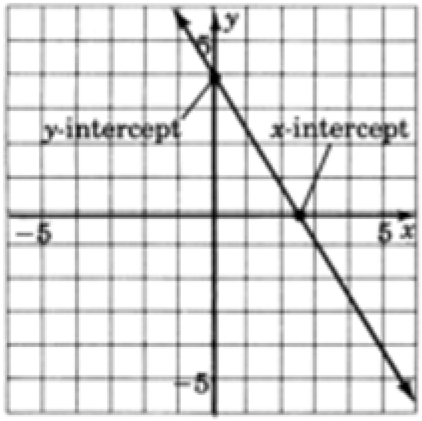
Any point (ordered pairs) that lies on the graph of a linear equation in two variables is a solution to that equation.

The Intercept Method of Graphing

When we want to graph a linear equation, it is certainly impractical to graph infinitely many points. Since a straight line is determined by only two points, we need only find two solutions to the equation (although a third point is helpful as a check).

Intercepts
When a linear equation in two variables is given in general from, $ax + by = c$, often the two most convenient points (solutions) to fine are called the **Intercepts:** these are the points at which the line intercepts the coordinate axes. Of course, a horizontal or vertical line intercepts only one

axis, so this method does not apply. Horizontal and vertical lines are easily recognized as they contain only **one** variable. (See Sample Set C.)



y-Intercept
The point at which the line crosses the y-axis is called the y-intercept. The x-value at this point is zero (since the point is neither to the left nor right of the origin).

x-Intercept
The point at which the line crosses the x-axis is called the x-intercept and the y-value at that point is zero. The y-intercept can be found by substituting the value 0 for x into the equation and solving for y. The x-intercept can be found by substituting the value 0 for y into the equation and solving for x.

Intercept Method
Since we are graphing an equation by finding the intercepts, we call this method the **intercept method**

Sample Set A

Graph the following equations using the intercept method.

Example:
$y - 2x = -3$

To find the y-intercept, let $x = 0$ and $y = b$.

$$\begin{aligned} b - 2(0) &= -3 \\ b - 0 &= -3 \\ b &= -3 \end{aligned}$$

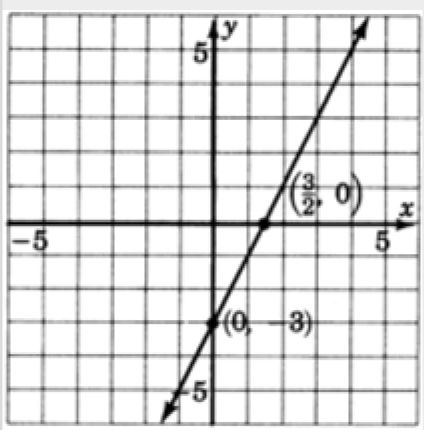
Thus, we have the point $(0, \ -3)$. So, if $x = 0$, $y = b = -3$.

To find the x-intercept, let $y = 0$ and $x = a$.

$$\begin{aligned} 0 - 2a &= -3 \\ -2a &= -3 \quad \text{Divide by -2.} \\ a &= \tfrac{-3}{-2} \\ a &= \tfrac{3}{2} \end{aligned}$$

Thus, we have the point $\left(\frac{3}{2}, 0\right)$. So, if $x = a = \frac{3}{2}$, $y = 0$.

Construct a coordinate system, plot these two points, and draw a line through them. Keep in mind that every point on this line is a solution to the equation $y - 2x = -3$.



Example:

$-2x + 3y = 3$

To find the y-intercept, let $x = 0$ and $y =$ b.

$$\begin{aligned} -2(0)+3b &= 3 \\ 0+3b &= 3 \\ 3b &= 3 \\ b &= 1 \end{aligned}$$

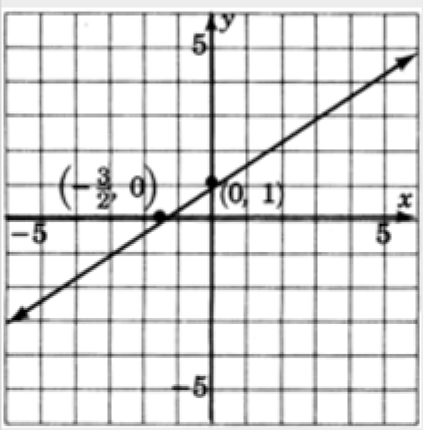
Thus, we have the point $(0,\ 1)$. So, if $x=0$, $y=b=1$.

To find the x-intercept, let $y=0$ and $x=a$.

$$\begin{aligned} -2a+3(0) &= 3 \\ -2a+0 &= 3 \\ -2a &= 3 \\ a &= \frac{3}{-2} \\ a &= -\frac{3}{2} \end{aligned}$$

Thus, we have the point $\left(-\frac{3}{2},\ 0\right)$. So, if $x=a=-\frac{3}{2}$, $y=0$.

Construct a coordinate system, plot these two points, and draw a line through them. Keep in mind that all the solutions to the equation $-2x+3y=3$ are precisely on this line.



Example:

$4x+y=5$

To find the y-intercept, let $x=0$ and $y=b$.

$$\begin{aligned}4(0)+b &= 5\\ 0+b &= 5\\ b &= 5\end{aligned}$$

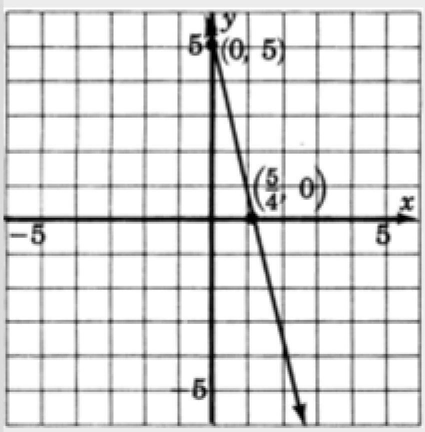
Thus, we have the point $(0,\,5)$. So, if $x=0$, $y=b=5$.

To find the x-intercept, let $y=0$ and $x=a$.

$$\begin{aligned}4a+0 &= 5\\ 4a &= 5\\ a &= \frac{5}{4}\end{aligned}$$

Thus, we have the point $\left(\frac{5}{4},\,0\right)$. So, if $x=a=\frac{5}{4}$, $y=0$.

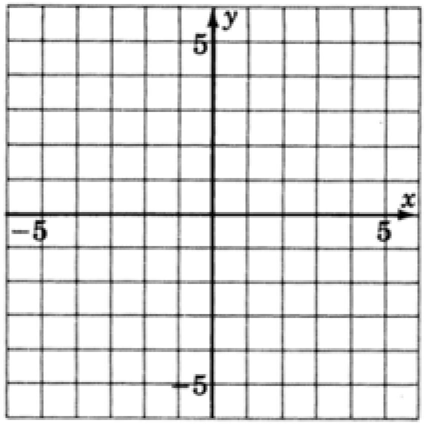
Construct a coordinate system, plot these two points, and draw a line through them.



Practice Set A

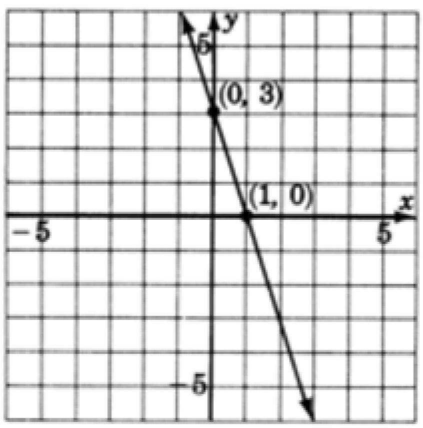
Exercise:

Problem: Graph $3x+y=3$ using the intercept method.



Solution:

When $x = 0$, $y = 3$; when $y = 0$, $x = 1$



Graphing Using any Two or More Points

The graphs we have constructed so far have been done by finding two particular points, the intercepts. Actually, **any** two points will do. We chose to use the intercepts because they are usually the easiest to work with. In the next example, we will graph two equations using points other than the intercepts. We'll use three points, the extra point serving as a check.

Sample Set B

Example:

$x - 3y = -10.$

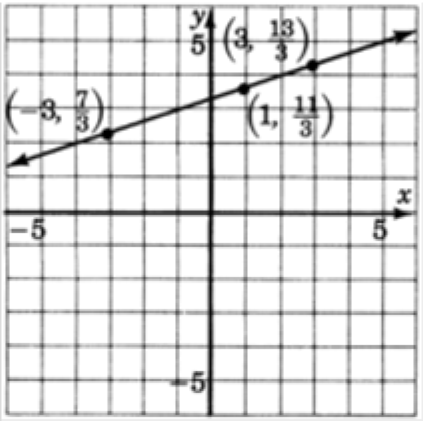
We can find three points by choosing three x-values and computing to find the corresponding y-values. We'll put our results in a table for ease of reading.

Since we are going to choose x-values and then compute to find the corresponding y-values, it will be to our advantage to solve the given equation for y.

$$\begin{aligned} x - 3y &= -10 && \text{Subtract } x \text{ from both sides.} \\ -3y &= -x - 10 && \text{Divide both sides by } -3. \\ y &= \tfrac{1}{3}x + \tfrac{10}{3} \end{aligned}$$

x	y	$(x,\ y)$
1	If $x = 1$, then $y = \frac{1}{3}(1) + \frac{10}{3} = \frac{1}{3} + \frac{10}{3} = \frac{11}{3}$	$\left(1,\ \frac{11}{3}\right)$
-3	If $x = -3$, then $y = \frac{1}{3}(-3) + \frac{10}{3} = -1 + \frac{10}{3} = \frac{7}{3}$	$\left(-3,\ \frac{7}{3}\right)$
3	If $x = 3$, then $y = \frac{1}{3}(3) + \frac{10}{3} = 1 + \frac{10}{3} = \frac{13}{3}$	$\left(3,\ \frac{13}{3}\right)$

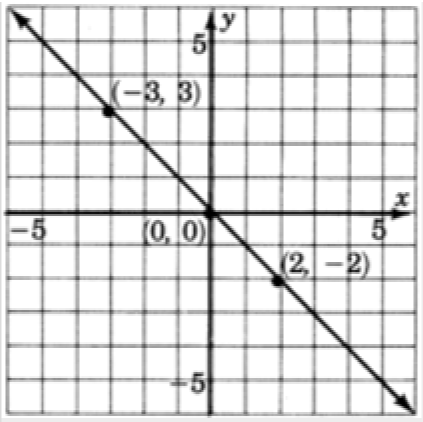
Thus, we have the three ordered pairs (points), $\left(1,\ \frac{11}{3}\right)$, $\left(-3,\ \frac{7}{3}\right)$, $\left(3,\ \frac{13}{3}\right)$. If we wish, we can change the improper fractions to mixed numbers, $\left(1, 3\,\frac{2}{3}\right)$, $\left(-3, 2\,\frac{1}{3}\right)$, $\left(3, 4\,\frac{1}{3}\right)$.



Example:
$4x + 4y = 0$
We solve for y.

$$\begin{aligned} 4y &= -4x \\ y &= -x \end{aligned}$$

x	y	(x, y)
0	0	$(0, 0)$
2	-2	$(2, -2)$
-3	3	$(-3, 3)$

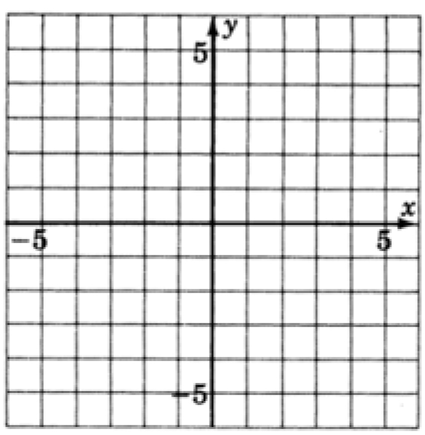


Notice that the $x-$ and y-intercepts are the same point. Thus the intercept method does not provide enough information to construct this graph.
When an equation is given in the general form $ax + by = c$, usually the most efficient approach to constructing the graph is to use the intercept method, when it works.

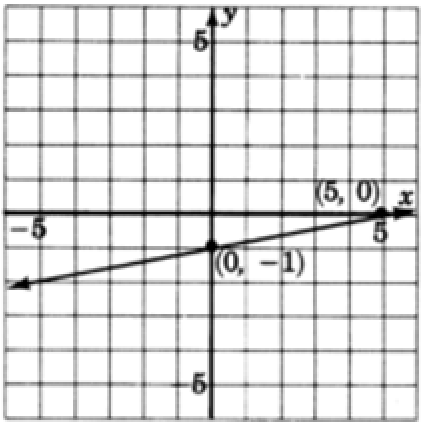
Practice Set B

Graph the following equations.
Exercise:

Problem: $x - 5y = 5$

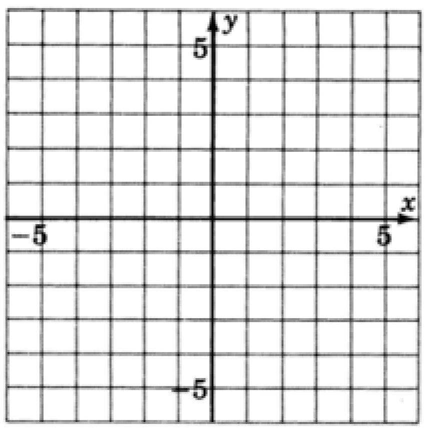


Solution:

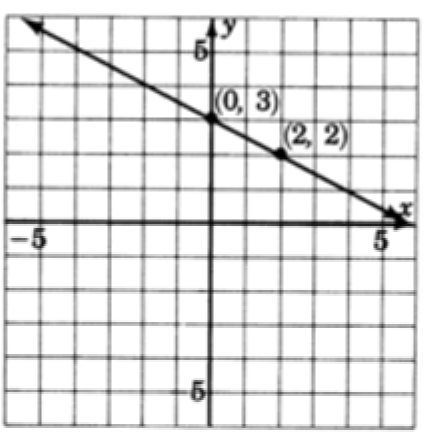


Exercise:

Problem: $x + 2y = 6$

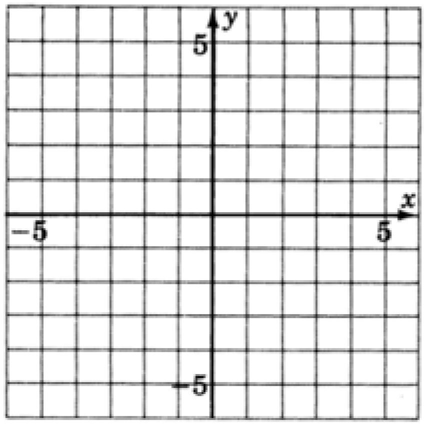


Solution:



Exercise:

Problem: $2x + y = 1$



Solution: [missing_resource: C06_S6-3_004.png]

Slanted, Horizontal, and Vertical Lines

In all the graphs we have observed so far, the lines have been slanted. This will always be the case when **both** variables appear in the equation. If only one variable appears in the equation, then the line will be either vertical or horizontal. To see why, let's consider a specific case:

Using the general form of a line, $ax + by = c$, we can produce an equation with exactly one variable by choosing $a = 0$, $b = 5$, and $c = 15$. The equation $ax + by = c$ then becomes

$0x + 5y = 15$

Since $0 \cdot (\text{any number}) = 0$, the term $0x$ is 0 for any number that is chosen for x.

Thus,

$0x + 5y = 15$

becomes

$0 + 5y = 15$

But, 0 is the additive identity and $0 + 5y = 5y$.

$5y = 15$

Then, solving for y we get

$y = 3$

This is an equation in which exactly one variable appears.

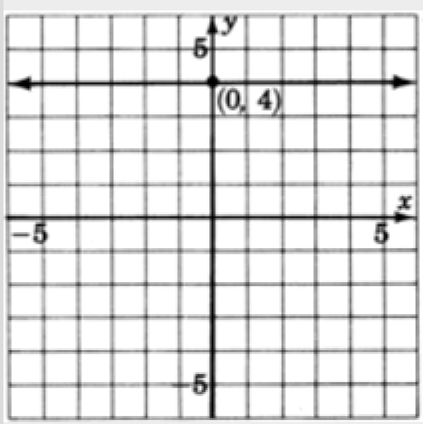
This means that regardless of which number we choose for x, the corresponding y-value is 3. Since the y-value is always the same as we move from left-to-right through the x-values, the height of the line above the x-axis is always the same (in this case, 3 units). This type of line must be horizontal.

An argument similar to the one above will show that if the only variable that appears is x, we can expect to get a vertical line.

Sample Set C

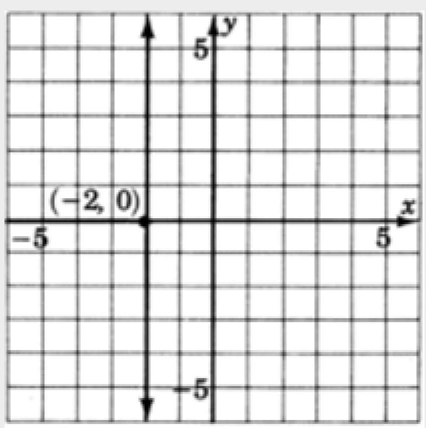
Example:
Graph $y = 4$.
The only variable appearing is y. Regardless of which x-value we choose, the y-value is always 4. All points with a y-value of 4 satisfy the equation. Thus we get a horizontal line 4 unit above the x-axis.

x	y	(x, y)
-3	4	$(-3, 4)$
-2	4	$(-2, 4)$
-1	4	$(-1, 4)$
0	4	$(0, 4)$
1	4	$(1, 4)$
2	4	$(2, 4)$
3	4	$(3, 4)$
4	4	$(4, 4)$



Example:
Graph $x = -2$.
The only variable that appears is x. Regardless of which y-value we choose, the x-value will always be -2. Thus, we get a vertical line two units to the left of the y-axis.

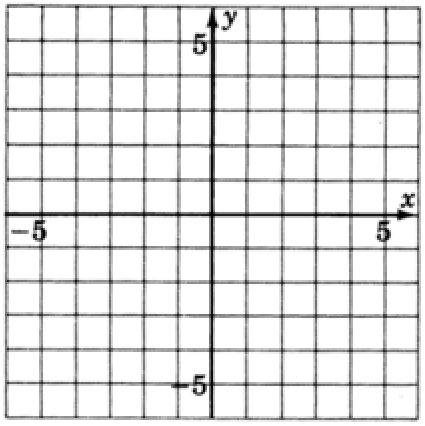
x	y	(x, y)
-2	-4	$(-2,\ -4)$
-2	-3	$(-2,\ -3)$
-2	-2	$(-2,\ -2)$
-2	-1	$(-2,\ -1)$
-2	0	$(-2, 0)$
-2	1	$(-2, 1)$
-2	2	$(-2, 0)$
-2	3	$(-2, 3)$
-2	4	$(-2, 4)$

- 

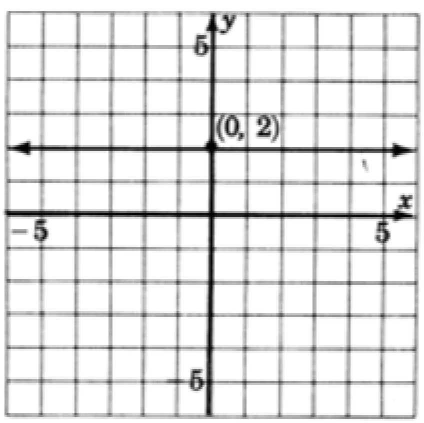
Practice Set C

Exercise:

Problem: Graph $y = 2$.

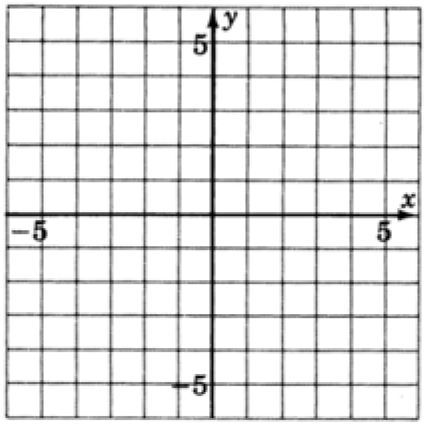


Solution:

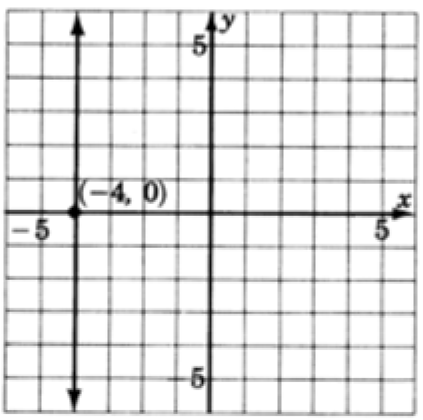


Exercise:

Problem: Graph $x = -4$.



Solution:



Summarizing our results we can make the following observations:

1. When a linear equation in two variables is written in the form $ax + by = c$, we say it is written in **general form**.
2. To graph an equation in general form it is sometimes convenient to use the intercept method.
3. A linear equation in which both variables appear will graph as a slanted line.
4. A linear equation in which only one variable appears will graph as either a vertical or horizontal line.

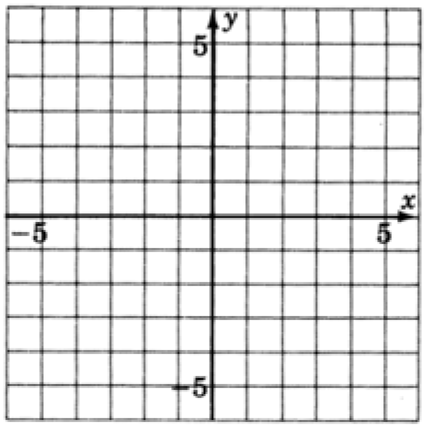
   $x = a$ graphs as a vertical line passing through a on the x-axis.
   $y = b$ graphs as a horizontal line passing through b on the y-axis.

Exercises

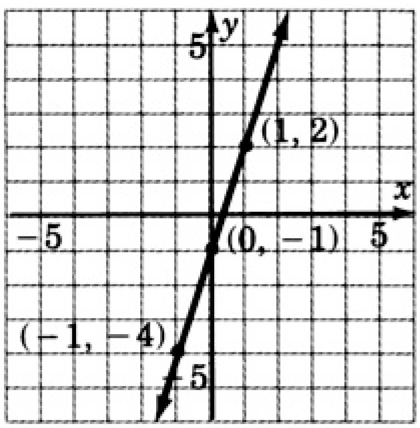
For the following problems, graph the equations.

Exercise:

Problem: $-3x + y = -1$

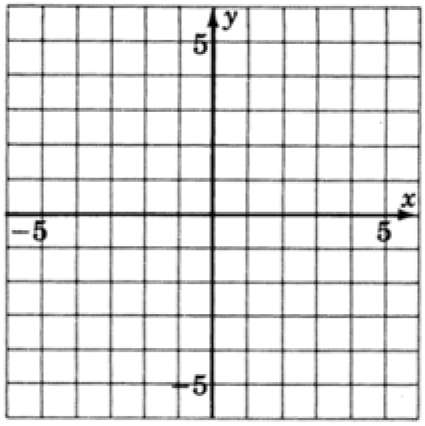


Solution:



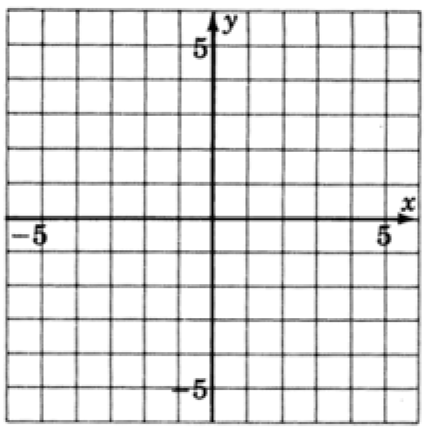
Exercise:

Problem: $3x - 2y = 6$

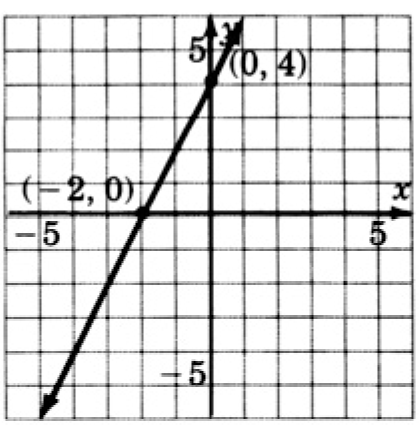


Exercise:

Problem: $-2x + y = 4$

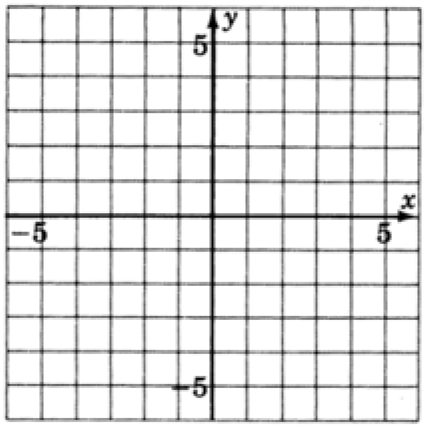


Solution:



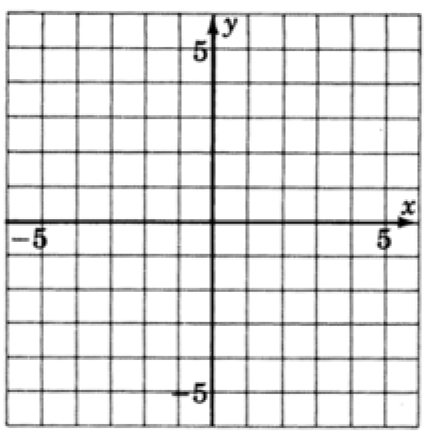
Exercise:

Problem: $x - 3y = 5$

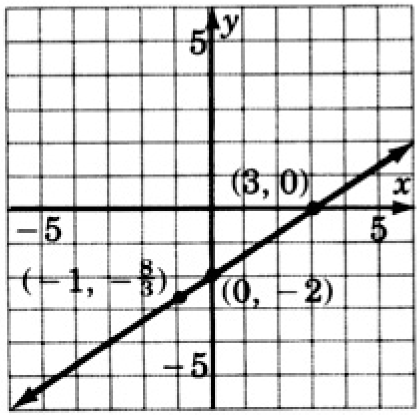


Exercise:

Problem: $2x - 3y = 6$

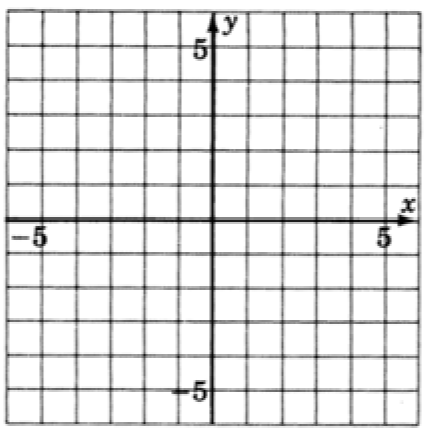


Solution:



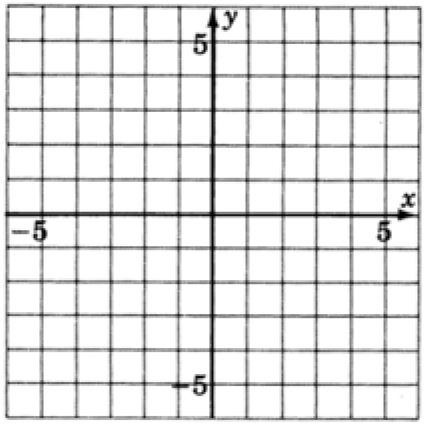
Exercise:

Problem: $2x + 5y = 10$

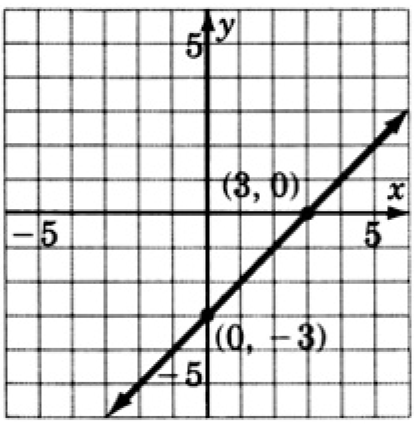


Exercise:

Problem: $3\,(x - y) = 9$

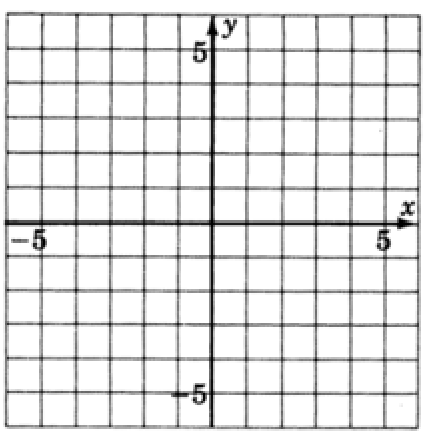


Solution:



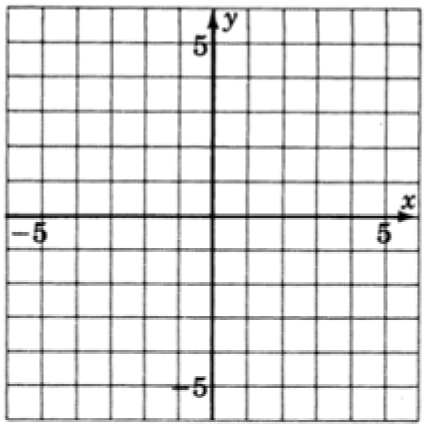
Exercise:

Problem: $-2x + 3y = -12$

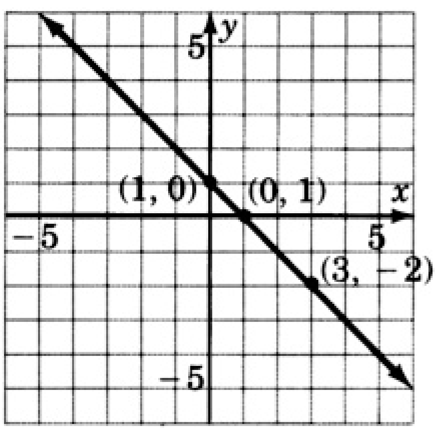


Exercise:

Problem: $y + x = 1$

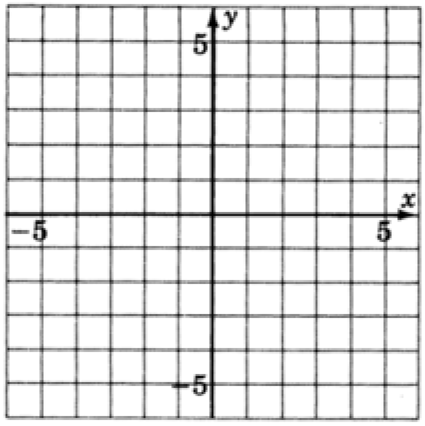


Solution:



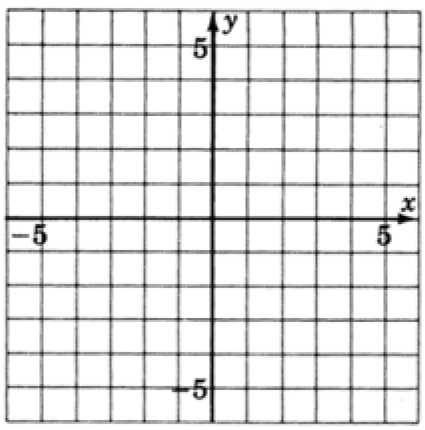
Exercise:

Problem: $4y - x - 12 = 0$

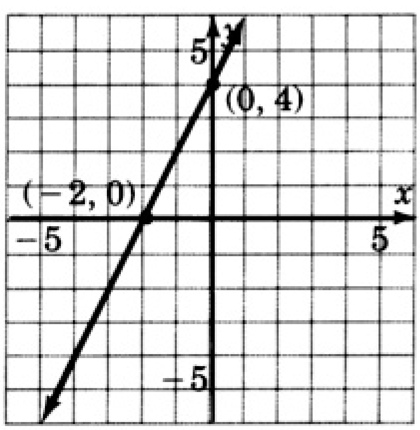


Exercise:

Problem: $2x - y + 4 = 0$

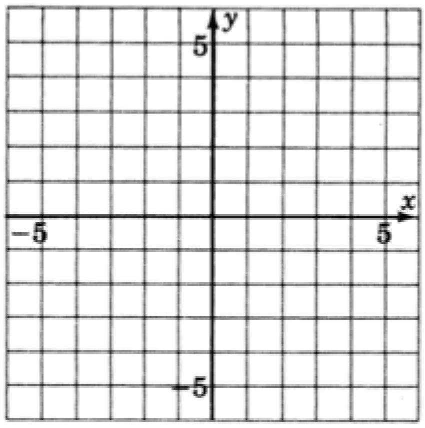


Solution:



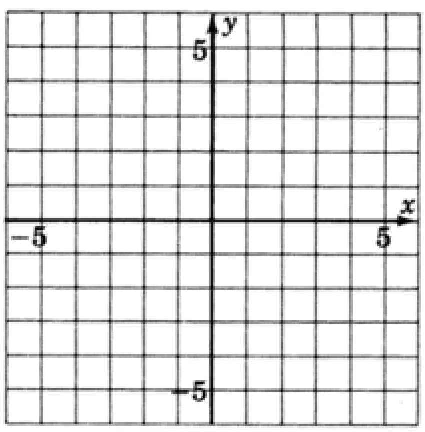
Exercise:

Problem: $-2x + 5y = 0$

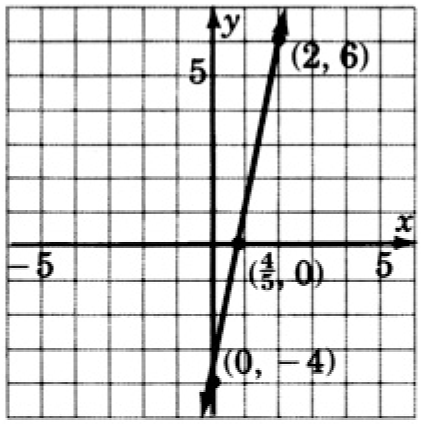


Exercise:

Problem: $y - 5x + 4 = 0$

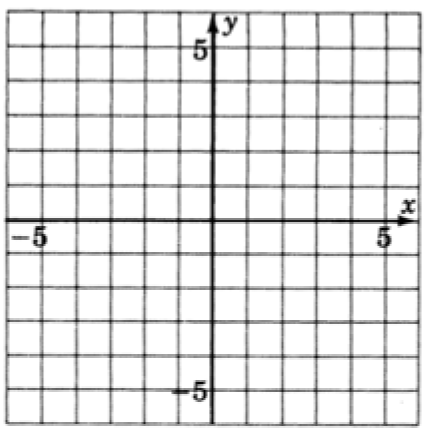


Solution:



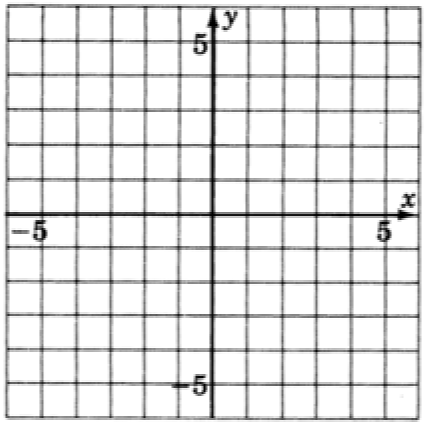
Exercise:

Problem: $0x + y = 3$

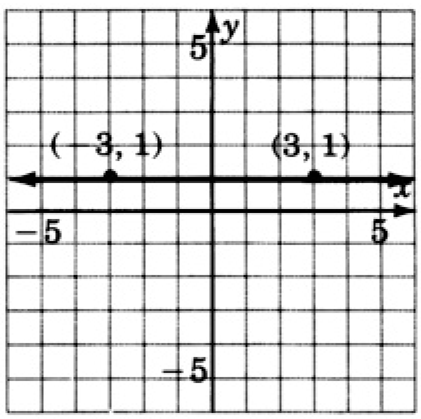


Exercise:

Problem: $0x + 2y = 2$

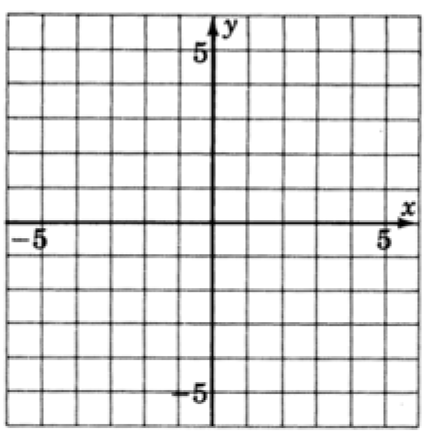


Solution:



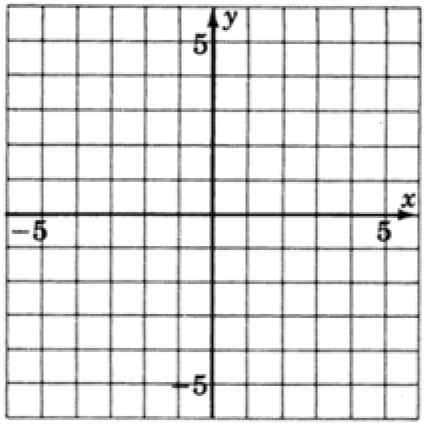
Exercise:

Problem: $0x + \frac{1}{4}y = 1$

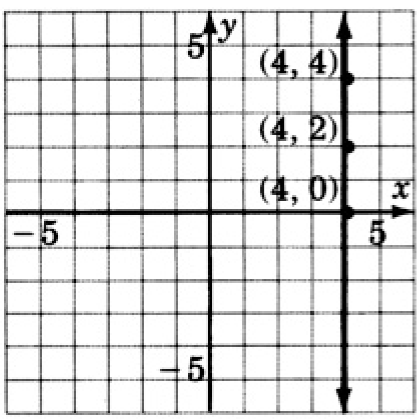


Exercise:

Problem: $4x + 0y = 16$

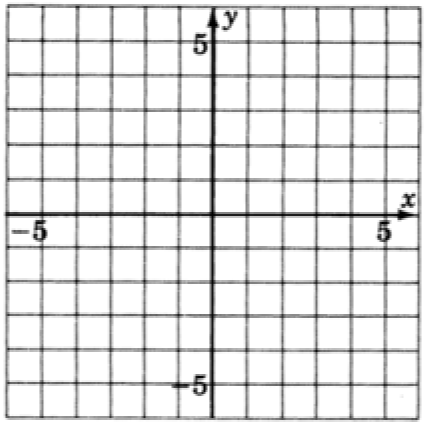


Solution:



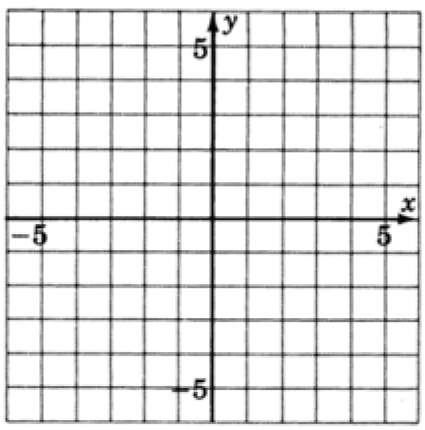
Exercise:

Problem: $\frac{1}{2}x + 0y = -1$



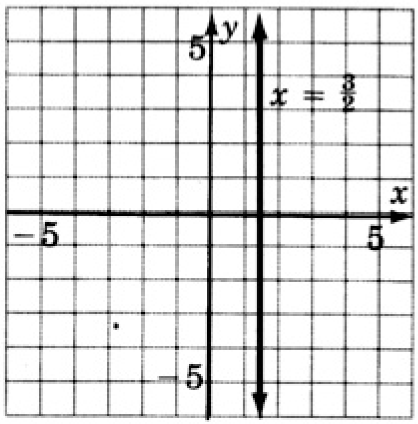
Exercise:

Problem: $\frac{2}{3}x + 0y = 1$



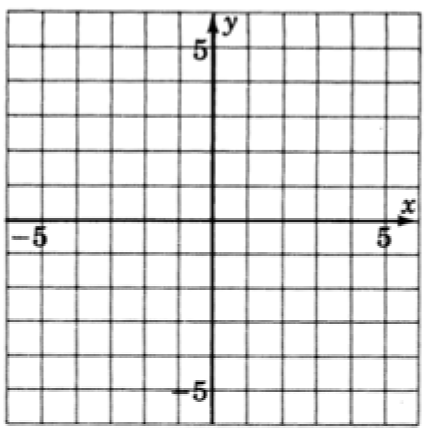
Solution:

$x = \frac{3}{2}$



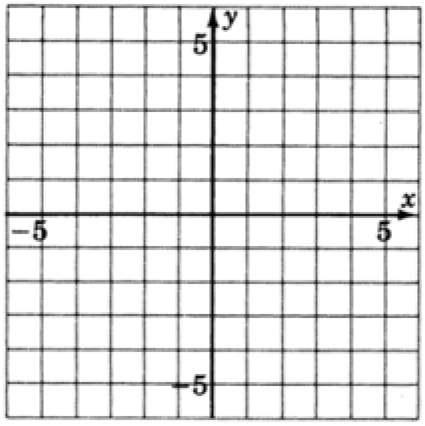
Exercise:

Problem: $y = 3$



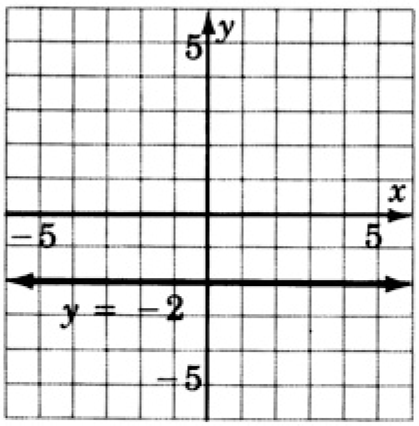
Exercise:

Problem: $y = -2$



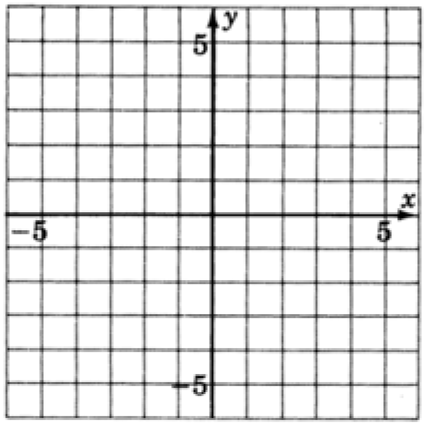
Solution:

$y=-2$



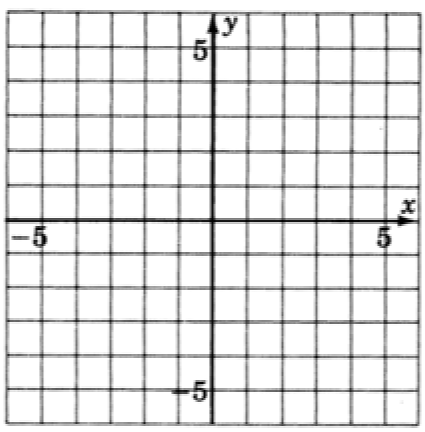
Exercise:

Problem: $-4y=20$

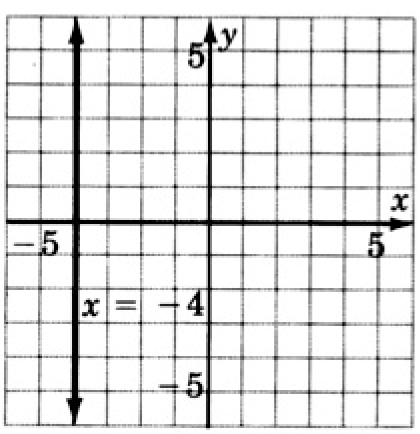


Exercise:

Problem: $x = -4$

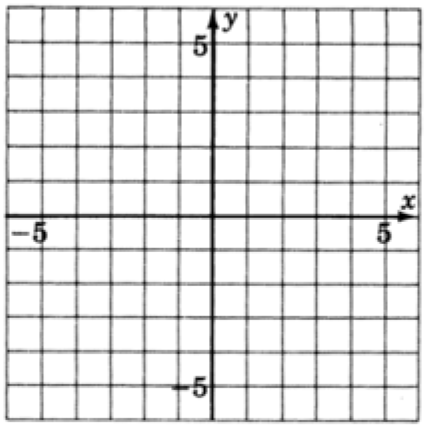


Solution:



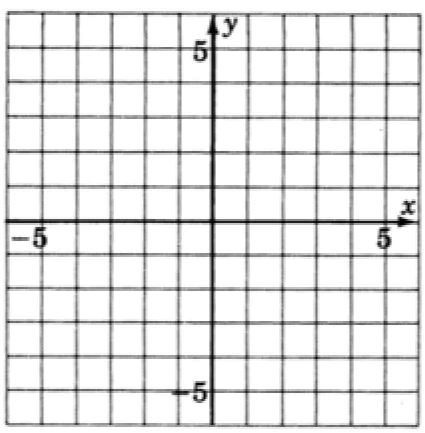
Exercise:

Problem: $-3x = -9$

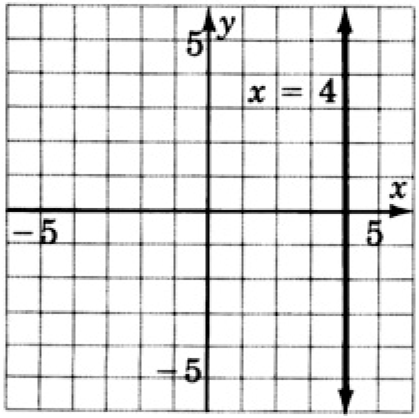


Exercise:

Problem: $-x + 4 = 0$



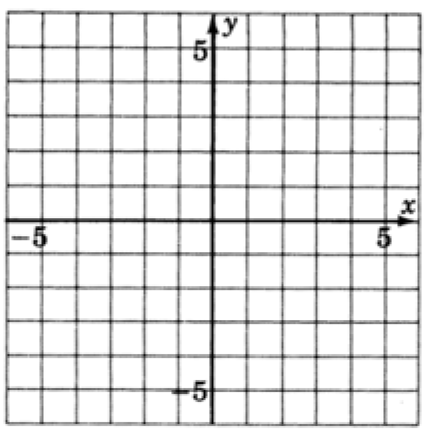
Solution:



Exercise:

Problem:

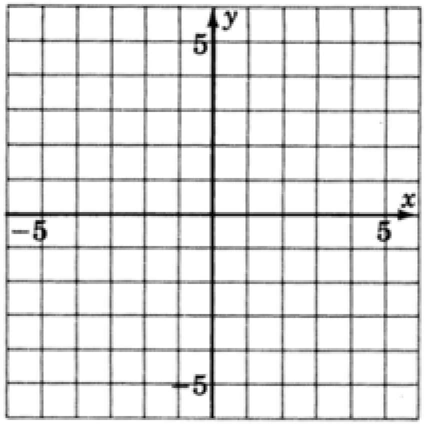
Construct the graph of all the points that have coordinates $(a,\ a)$, that is, for each point, the $x-$ and y-values are the same.



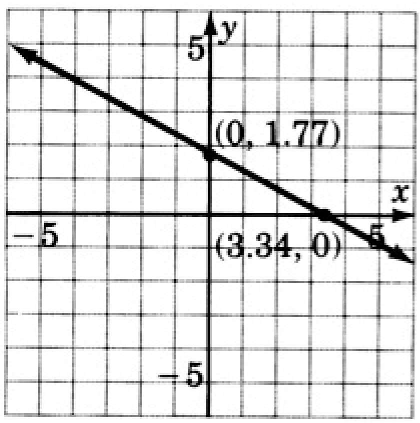
Calculator Problems

Exercise:

Problem: $2.53x + 4.77y = 8.45$

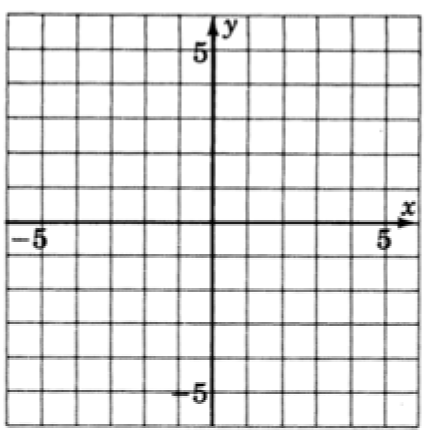


Solution:



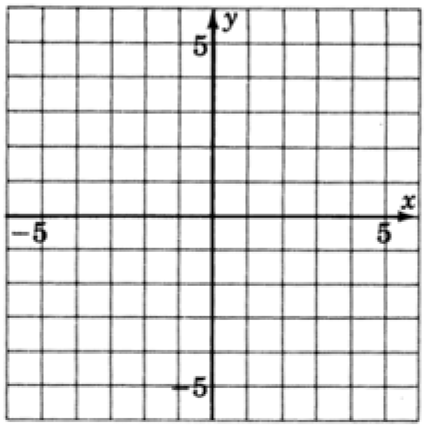
Exercise:

Problem: $1.96x + 2.05y = 6.55$

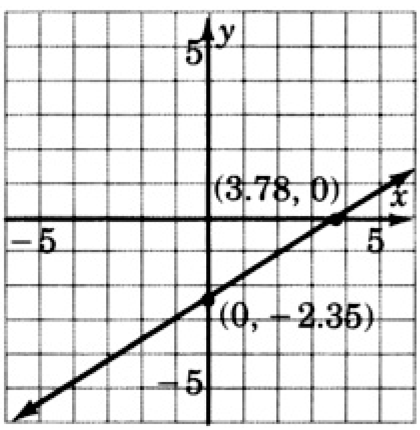


Exercise:

Problem: $4.1x - 6.6y = 15.5$

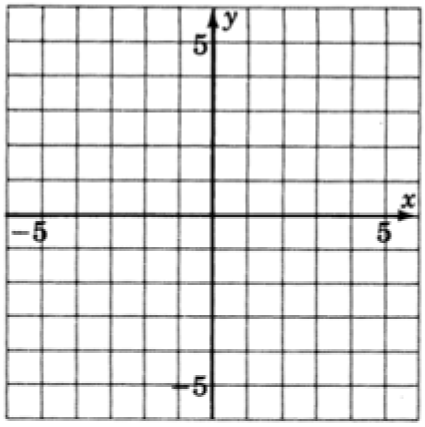


Solution:



Exercise:

Problem: $626.01x - 506.73y = 2443.50$



Exercises for Review

Exercise:

Problem:

(**[link]**) Name the property of real numbers that makes $4 + x = x + 4$ a true statement.

Solution:

commutative property of addition

Exercise:

Problem:

(**[link]**) Supply the missing word. The absolute value of a number a, denoted $|a|$, is the from a to 0 on the number line.

Exercise:

Problem: (**[link]**) Find the product $(3x + 2)(x - 7)$.

Solution:

$3x^2 - 19x - 14$

Exercise:

Problem: ([link]) Solve the equation $3\left[3\left(x-2\right)+4x\right]-24=0$.

Exercise:

Problem:

([link]) Supply the missing word. The coordinate axes divide the plane into four equal regions called .

Solution:

quadrants

Solving Systems of Linear Equations by Elimination
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Beginning with the graphical solution of systems, this chapter includes an interpretation of independent, inconsistent, and dependent systems and examples to illustrate the applications for these systems. The substitution method and the addition method of solving a system by elimination are explained, noting when to use each method. The five-step method is again used to illustrate the solutions of value and rate problems (coin and mixture problems), using drawings that correspond to the actual situation. Objectives of this module: know the properties used in the addition method, be able to use the addition method to solve a system of linear equations, know what to expect when using the addition method with a system that consists of parallel or coincident lines.

Overview

- The Properties Used in the Addition Method
- The Addition Method
- Addition and Parallel or Coincident Lines

The Properties Used in the Addition Method

Another method of solving a system of two linear equations in two variables is called the **method of elimination by addition**. It is similar to the method of elimination by substitution in that the process eliminates one equation and one variable. The method of elimination by addition makes use of the following two properties.

1. If A, B, and C are algebraic expressions such that

$$\begin{array}{rcl} A & = & B \\ C & = & D \\ \hline A+C & = & B+D \end{array} \quad \begin{array}{l} \text{and} \\ \text{then} \end{array}$$

2. $ax + (-ax) = 0$

Property 1 states that if we add the left sides of two equations together and the right sides of the same two equations together, the resulting sums will be equal. We call this **adding equations**. Property 2 states that the sum of two opposites is zero.

The Addition Method

To solve a system of two linear equations in two variables by addition,

1. Write, if necessary, both equations in general form, $ax + by = c$.
2. If necessary, multiply one or both equations by factors that will produce opposite coefficients for one of the variables.
3. Add the equations to eliminate one equation and one variable.
4. Solve the equation obtained in step 3.
5. Do one of the following:
   (a) Substitute the value obtained in step 4 into either of the original equations and solve to obtain the value of the other variable,
   or
   (b) Repeat steps 1-5 for the other variable.
6. Check the solutions in both equations.
7. Write the solution as an ordered pair.

The addition method works well when the coefficient of one of the variables is 1 or a number other than 1.

Sample Set A

Example:

Solve $\begin{cases} x - y = 2 & (1) \\ 3x + y = 14 & (2) \end{cases}$

Step 1: Both equations appear in the proper form.

Step 2: The coefficients of y are already opposites, 1 and -1, so there is no need for a multiplication.

Step 3: Add the equations.

$$\begin{array}{r} x-y=2 \\ \underline{3x+y=14} \\ 4x+0=16 \end{array}$$

Step 4: Solve the equation $4x = 16$.

$4x = 16$

$x = 4$

The problem is not solved yet; we still need the value of y.

Step 5: Substitute $x = 4$ into either of the original equations. We will use equation 1.

$$\begin{aligned} 4 - y &= 2 \quad \text{Solve for } y. \\ -y &= -2 \\ y &= 2 \end{aligned}$$

We now have $x = 4$, $y = 2$.

Step 6: Substitute $x = 4$ and $y = 2$ into both the original equations for a check.

$$\begin{array}{llcll} (1) & x - y & = & 2 & \\ & 4 - 2 & = & 2 & \text{Is this correct?} \\ & 2 & = & 2 & \text{Yes, this is correct.} \end{array} \qquad \begin{array}{llcll} (2) & 3x + y & = & 14 & \\ & 3(4) + 2 & = & 14 & \text{Is this correct?} \\ & 12 + 2 & = & 14 & \text{Is this correct?} \\ & 14 & = & 14 & \text{Yes, this is correct.} \end{array}$$

Step 7: The solution is $(4, 2)$.

The two lines of this system intersect at $(4, 2)$.

Practice Set A

Solve each system by addition.

Exercise:

Problem: $\begin{cases} x + y = 6 \\ 2x - y = 0 \end{cases}$

Solution:

$(2, 4)$

Exercise:

Problem: $\begin{cases} x + 6y = 8 \\ -x - 2y = 0 \end{cases}$

Solution:

$(-4, 2)$

Sample Set B

Solve the following systems using the addition method.

Example:

Solve $\begin{cases} 6a - 5b = 14 & (1) \\ 2a + 2b = -10 & (2) \end{cases}$

Step 1: The equations are already in the proper form, $ax + by = c$.

Step 2: If we multiply equation (2) by —3, the coefficients of a will be opposites and become 0 upon addition, thus eliminating a.

$$\begin{cases} 6a - 5b = 14 \\ -3\left(2a + 2b\right) = -3\left(10\right) \end{cases} \quad \rightarrow \quad \begin{cases} 6a - 5b = 14 \\ -6a - 6b = 30 \end{cases}$$

Step 3: Add the equations.

$$\begin{array}{r} 6a - 5b = 14 \\ \underline{-6a - 6b = 30} \\ 0 - 11b = 44 \end{array}$$

Step 4: Solve the equation $-11b = 44$.

$$\begin{aligned} -11b &= 44 \\ b &= -4 \end{aligned}$$

Step 5: Substitute$b = -4$ into either of the original equations. We will use equation 2.

$$\begin{aligned} 2a+2b &= -10 \\ 2a+2(-4) &= -10 \quad \text{Solve for } a. \\ 2a-8 &= -10 \\ 2a &= -2 \\ a &= -1 \end{aligned}$$

We now have $a=-1$ and $b=-4$.

Step 6: Substitute $a=-1$ and $b=-4$ into both the original equations for a check.

$$(1) \quad \begin{aligned} 6a-5b &= 14 \\ 6(-1)-5(-4) &= 14 \quad \text{Is this correct?} \\ -6+20 &= 14 \quad \text{Is this correct?} \\ 14 &= 14 \quad \text{Yes, this is correct.} \end{aligned}$$

$$(2) \quad \begin{aligned} 2a+2b &= -10 \\ 2(-1)+2(-4) &= -10 \quad \text{Is this correct?} \\ -2-8 &= -10 \quad \text{Is this correct?} \\ -10 &= -10 \quad \text{Yes, this is correct.} \end{aligned}$$

Step 7: The solution is $(-1,-4)$.

Example:

Solve $\begin{cases} 3x+2y=-4 & (1) \\ 4x=5y+10 & (2) \end{cases}$

Step 1: Rewrite the system in the proper form.

$$\begin{cases} 3x+2y=-4 & (1) \\ 4x-5y=10 & (2) \end{cases}$$

Step 2: Since the coefficients of y already have opposite signs, we will eliminate y.
Multiply equation (1) by 5, the coefficient of y in equation 2.
Multiply equation (2) by 2, the coefficient of y in equation 1.

$$\begin{cases} 5(3x+2y)=5(-4) \\ 2(4x-5y)=2(10) \end{cases} \quad \rightarrow \quad \begin{cases} 15x+10y=-20 \\ 8x-10y=20 \end{cases}$$

Step 3: Add the equations.

$$\frac{\begin{array}{c} 15x+10y=-20 \\ 8x-10y=20 \end{array}}{23x+0=0}$$

Step 4: Solve the equation $23x=0$

$$23x=0$$

$$x=0$$

Step 5: Substitute $x=0$ into either of the original equations. We will use equation 1.

$$\begin{array}{rcll} 3x+2y & = & -4 & \\ 3\left(0\right)+2y & = & -4 & \text{Solve for } y. \\ 0+2y & = & -4 & \\ y & = & -2 & \end{array}$$

We now have $x=0$ and $y=-2$.

Step 6: Substitution will show that these values check.

Step 7: The solution is $(0,-2)$.

Practice Set B

Solve each of the following systems using the addition method.

Exercise:

Problem: $\begin{cases} 3x+y=1 \\ 5x+y=3 \end{cases}$

Solution:

$(1,-2)$

Exercise:

Problem: $\begin{cases} x+4y=1 \\ x-2y=-5 \end{cases}$

Solution:

$(-3,1)$

Exercise:

Problem: $\begin{cases} 2x+3y=-10 \\ -x+2y=-2 \end{cases}$

Solution:

$(-2,-2)$

Exercise:

Problem: $\begin{cases} 5x-3y=1 \\ 8x-6y=4 \end{cases}$

Solution:

$(-1,-2)$

Exercise:

Problem: $\begin{cases} 3x - 5y = 9 \\ 4x + 8y = 12 \end{cases}$

Solution:

$(3, 0)$

Addition And Parallel Or Coincident Lines

When the lines of a system are parallel or coincident, the method of elimination produces results identical to that of the method of elimination by substitution.

Addition and Parallel Lines
If computations eliminate all variables and produce a contradiction, the two lines of the system are parallel and the system is called inconsistent.

Addition and Coincident Lines
If computations eliminate all variables and produce an identity, the two lines of the system are coincident and the system is called dependent.

Sample Set C

Example:
Solve $\begin{cases} 2x - y = 1 & (1) \\ 4x - 2y = 4 & (2) \end{cases}$

Step 1: The equations are in the proper form.

Step 2: We can eliminate x by multiplying equation (1) by –2.

$$\begin{cases} -2(2x - y) = -2(1) \\ 4x - 2y = 4 \end{cases} \quad \rightarrow \quad \begin{cases} -4x + 2y = -2 \\ 4x - 2y = 4 \end{cases}$$

Step 3: Add the equations.

$$\begin{array}{c} -4x+2y=-2 \\ \underline{4x-2y=4} \\ 0+0=2 \\ 0=2 \end{array}$$

This is false and is therefore a contradiction. The lines of this system are parallel. This system is inconsistent.

Example:
Solve $\begin{cases} 4x + 8y = 8 & (1) \\ 3x + 6y = 6 & (2) \end{cases}$

Step 1: The equations are in the proper form.

Step 2: We can eliminate x by multiplying equation (1) by –3 and equation (2) by 4.

$$\begin{cases} -3\left(4x+8y\right)=-3\left(8\right) \\ 4\left(3x+6y\right)=4\left(6\right) \end{cases} \rightarrow \begin{cases} -12x-24y=-24 \\ 12x+24y=24 \end{cases}$$

Step 3: Add the equations.

$$\begin{array}{r} -12x-24y=-24 \\ \underline{12x+24y=24} \\ 0+0=0 \\ 0=0 \end{array}$$

This is true and is an identity. The lines of this system are coincident.

This system is dependent.

Practice Set C

Solve each of the following systems using the addition method.
Exercise:

Problem: $\begin{cases} -x+2y=6 \\ -6x+12y=1 \end{cases}$

Solution:

inconsistent

Exercise:

Problem: $\begin{cases} 4x-28y=-4 \\ x-7y=-1 \end{cases}$

Solution:

dependent

Exercises

For the following problems, solve the systems using elimination by addition.
Exercise:

Problem: $\begin{cases} x+y=11 \\ x-y=-1 \end{cases}$

Solution:

$(5,6)$

Exercise:

Problem: $\begin{cases} x+3y=13 \\ x-3y=-11 \end{cases}$

Exercise:

Problem: $\begin{cases} 3x-5y=-4 \\ -4x+5y=2 \end{cases}$

Solution:

$(2,2)$

Exercise:

Problem: $\begin{cases} 2x-7y=1 \\ 5x+7y=-22 \end{cases}$

Exercise:

Problem: $\begin{cases} -3x+4y=-24 \\ 3x-7y=42 \end{cases}$

Solution:

$(0,-6)$

Exercise:

Problem: $\begin{cases} 8x+5y=3 \\ 9x-5y=-71 \end{cases}$

Exercise:

Problem: $\begin{cases} -x+2y=-6 \\ x+3y=-4 \end{cases}$

Solution:

$(2,-2)$

Exercise:

Problem: $\begin{cases} 4x+y=0 \\ 3x+y=0 \end{cases}$

Exercise:

Problem: $\begin{cases} x+y=-4 \\ -x-y=4 \end{cases}$

Solution:

dependent

Exercise:

Problem: $\begin{cases} -2x-3y=-6 \\ 2x+3y=6 \end{cases}$

Exercise:

Problem: $\begin{cases} 3x + 4y = 7 \\ x + 5y = 6 \end{cases}$

Solution:

$(1, 1)$

Exercise:

Problem: $\begin{cases} 4x - 2y = 2 \\ 7x + 4y = 26 \end{cases}$

Exercise:

Problem: $\begin{cases} 3x + y = -4 \\ 5x - 2y = -14 \end{cases}$

Solution:

$(-2, 2)$

Exercise:

Problem: $\begin{cases} 5x - 3y = 20 \\ -x + 6y = -4 \end{cases}$

Exercise:

Problem: $\begin{cases} 6x + 2y = -18 \\ -x + 5y = 19 \end{cases}$

Solution:

$(-4, 3)$

Exercise:

Problem: $\begin{cases} x - 11y = 17 \\ 2x - 22y = 4 \end{cases}$

Exercise:

Problem: $\begin{cases} -2x + 3y = 20 \\ -3x + 2y = 15 \end{cases}$

Solution:

$(-1, 6)$

Exercise:

Problem: $\begin{cases} -5x + 2y = -4 \\ -3x - 5y = 10 \end{cases}$

Exercise:

Problem: $\begin{cases} -3x - 4y = 2 \\ -9x - 12y = 6 \end{cases}$

Solution:

dependent

Exercise:

Problem: $\begin{cases} 3x - 5y = 28 \\ -4x - 2y = -20 \end{cases}$

Exercise:

Problem: $\begin{cases} 6x - 3y = 3 \\ 10x - 7y = 3 \end{cases}$

Solution:

$(1,\ 1)$

Exercise:

Problem: $\begin{cases} -4x + 12y = 0 \\ -8x + 16y = 0 \end{cases}$

Exercise:

Problem: $\begin{cases} 3x + y = -1 \\ 12x + 4y = 6 \end{cases}$

Solution:

inconsistent

Exercise:

Problem: $\begin{cases} 8x + 5y = -23 \\ -3x - 3y = 12 \end{cases}$

Exercise:

Problem: $\begin{cases} 2x + 8y = 10 \\ 3x + 12y = 15 \end{cases}$

Solution:

dependent

Exercise:

Problem: $\begin{cases} 4x + 6y = 8 \\ 6x + 8y = 12 \end{cases}$

Exercise:

Problem: $\begin{cases} 10x + 2y = 2 \\ -15x - 3y = 3 \end{cases}$

Solution:

inconsistent

Exercise:

Problem: $\begin{cases} x + \frac{3}{4}y = -\frac{1}{2} \\ \frac{3}{5}x + y = -\frac{7}{5} \end{cases}$

Exercise:

Problem: $\begin{cases} x + \frac{1}{3}y = \frac{4}{3} \\ -x + \frac{1}{6}y = \frac{2}{3} \end{cases}$

Solution:

$(0, 4)$

Exercise:

Problem: $\begin{cases} 8x - 3y = 25 \\ 4x - 5y = -5 \end{cases}$

Exercise:

Problem: $\begin{cases} -10x - 4y = 72 \\ 9x + 5y = 39 \end{cases}$

Solution:

$\left(-\frac{258}{7}, \frac{519}{7}\right)$

Exercise:

Problem: $\begin{cases} 12x + 16y = -36 \\ -10x + 12y = 30 \end{cases}$

Exercise:

Problem: $\begin{cases} 25x - 32y = 14 \\ -50x + 64y = -28 \end{cases}$

Solution:

dependent

Exercises For Review

Exercise:

Problem:([link]) Simplify and write $\left(2x^{-3}y^{4}\right)^{5}\left(2xy^{-6}\right)^{-5}$ so that only positive exponents appear.

Exercise:

Problem:([link]) Simplify $\sqrt{8}+3\sqrt{50}$.

Solution:

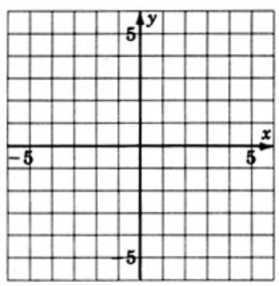
$17\sqrt{2}$

Exercise:

Problem:([link]) Solve the radical equation $\sqrt{2x+3}+5=8$.

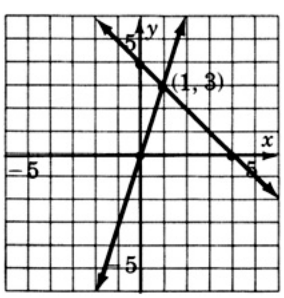
Exercise:

Problem:([link]) Solve by graphing $\begin{cases} x+y=4 \\ 3x-y=0 \end{cases}$



Solution:

$(1,3)$



Exercise:

Problem:([link]) Solve using the substitution method: $\begin{cases} 3x-4y=-11 \\ 5x+y=-3 \end{cases}$

Solving Systems of Linear Equations by Substitution
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Beginning with the graphical solution of systems, this chapter includes an interpretation of independent, inconsistent, and dependent systems and examples to illustrate the applications for these systems. The substitution method and the addition method of solving a system by elimination are explained, noting when to use each method. The five-step method is again used to illustrate the solutions of value and rate problems (coin and mixture problems), using drawings that correspond to the actual situation. Objectives of this module: know when the substitution method works best, be able to use the substitution method to solve a system of linear equations, know what to expect when using substitution with a system that consists of parallel lines.

Overview

- When Substitution Works Best
- The Substitution Method
- Substitution and Parallel Lines
- Substitution and Coincident Lines

When Substitution Works Best

We know how to solve a linear equation in one variable. We shall now study a method for solving a system of two linear equations in two variables by transforming the two equations in two variables into one equation in one variable.

To make this transformation, we need to eliminate one equation and one variable. We can make this **elimination by substitution**.

When Substitution Works Best
The substitution method works best when **either** of these conditions exists:

1. One of the variables has a coefficient of 1, or
2. One of the variables can be made to have a coefficient of 1 without introducing fractions.

The Substitution Method

The Substitution Method
To solve a system of two linear equations in two variables,

1. Solve one of the equations for one of the variables.
2. Substitute the expression for the variable chosen in step 1 into the other equation.
3. Solve the resulting equation in one variable.
4. Substitute the value obtained in step 3 into the equation obtained in step 1 and solve to obtain the value of the other variable.
5. Check the solution in both equations.
6. Write the solution as an ordered pair.

Sample Set A

Example:

Solve the system $\begin{cases} 2x + 3y = 14 & (1) \\ 3x + y = 7 & (2) \end{cases}$

Step 1: Since the coefficient of y in equation 2 is 1, we will solve equation 2 for y.

$$y = -3x + 7$$

Step 2: Substitute the expression $-3x + 7$ for y in equation 1.

$$2x + 3\left(-3x + 7\right) = 14$$

Step 3: Solve the equation obtained in step 2.

$$\begin{aligned} 2x + 3\left(-3x + 7\right) &= 14 \\ 2x - 9x + 21 &= 14 \\ -7x + 21 &= 14 \\ -7x &= -7 \\ x &= 1 \end{aligned}$$

Step 4: Substitute $x = 1$ into the equation obtained in step 1, $y = -3x + 7$.

$$\begin{aligned} y &= -3\left(1\right) + 7 \\ y &= -3 + 7 \\ y &= 4 \end{aligned}$$

We now have $x = 1$ and $y = 4$.

Step 5: Substitute $x = 1, y = 4$ into each of the original equations for a check.

$$(1) \quad \begin{aligned} 2x + 3y &= 14 \\ 2(1) + 3(4) &= 14 \quad \text{Is this correct?} \\ 2 + 12 &= 14 \quad \text{Is this correct?} \\ 14 &= 14 \quad \text{Yes, this is correct.} \end{aligned} \qquad (2) \quad \begin{aligned} 3x + y &= 7 \\ 3(1) + (4) &= 7 \quad \text{Is this correct?} \\ 3 + 4 &= 7 \quad \text{Is this correct?} \\ 7 &= 7 \quad \text{Yes, this is correct.} \end{aligned}$$

Step 6: The solution is $(1, 4)$. The point $(1, 4)$ is the point of intersection of the two lines of the system.

Practice Set A

Exercise:

Problem: Slove the system $\begin{cases} 5x - 8y = 18 \\ 4x + \ \ y = \ \ 7 \end{cases}$

Solution:

The point $(2,-1)$ is the point of intersection of the two lines.

Substitution And Parallel Lines

The following rule alerts us to the fact that the two lines of a system are parallel.

Substitution and Parallel Lines
If computations eliminate all the variables and produce a contradiction, the two lines of a system are parallel, and the system is called inconsistent.

Sample Set B

Example:
Solve the system $\begin{cases} 2x-y=1 & (1) \\ 4x-2y=4 & (2) \end{cases}$

Step 1: Solve equation 1 for y.

$$\begin{aligned} 2x-y &= 1 \\ -y &= -2x+1 \\ y &= 2x-1 \end{aligned}$$

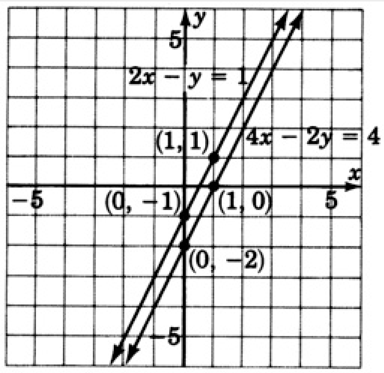
Step 2: Substitute the expression $2x-1$ for y into equation 2.

$$4x-2(2x-1)=4$$

Step 3: Solve the equation obtained in step 2.

$$\begin{aligned} 4x-2(2x-1) &= 4 \\ 4x-4x+2 &= 4 \\ 2 &\neq 4 \end{aligned}$$

Computations have eliminated all the variables and produce a contradiction. These lines are parallel.



This system is inconsistent.

Practice Set B

Exercise:

Problem: Slove the system $\begin{cases} 7x - 3y = 2 \\ 14x - 6y = 1 \end{cases}$

Solution:

Substitution produces $4 \neq 1$, or $\frac{1}{2} \neq 2$, a contradiction. These lines are parallel and the system is inconsistent.

Substitution And Coincident Lines

The following rule alerts us to the fact that the two lines of a system are coincident.

Substitution and Coincident Lines
If computations eliminate all the variables and produce an identity, the two lines of a system are coincident and the system is called dependent.

Sample Set C

Example:

Solve the system $\begin{cases} 4x + 8y = 8 & (1) \\ 3x + 6y = 6 & (2) \end{cases}$

Step 1: Divide equation 1 by 4 and solve for x.

$$\begin{aligned} 4x + 8y &= 8 \\ x + 2y &= 2 \\ x &= -2y + 2 \end{aligned}$$

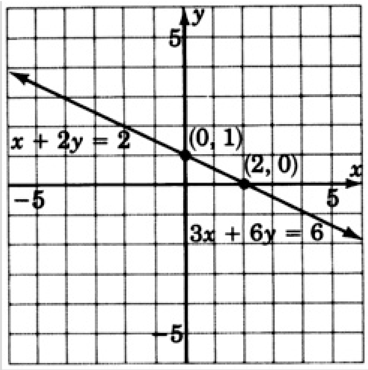
Step 2: Substitute the expression $-2y + 2$ for x in equation 2.

$$3(-2y + 2) + 6y = 6$$

Step 3: Solve the equation obtained in step 2.

$$\begin{aligned} 3(-2y + 2) + 6y &= 6 \\ -6y + 6 + 6y &= 6 \\ 6 &= 6 \end{aligned}$$

Computations have eliminated all the variables and produced an identity. These lines are coincident.



This system is dependent.

Practice Set C

Exercise:

Problem: Solve the system $\begin{cases} 4x+3y= & 1 \\ -8x-6y= & -2 \end{cases}$

Solution:

Computations produce $-2=-2$, an identity. These lines are coincident and the system is dependent.

Systems in which a coefficient of one of the variables is not 1 or cannot be made to be 1 without introducing fractions are not well suited for the substitution method. The problem in Sample Set D illustrates this “messy” situation.

Sample Set D

Example:

Solve the system $\begin{cases} 3x+2y=1 & (1) \\ 4x-3y=3 & (2) \end{cases}$

Step 1: We will solve equation (1) for y.

$$\begin{aligned} 3x+2y &= 1 \\ 2y &= -3x+1 \\ y &= \tfrac{-3}{2}x+\tfrac{1}{2} \end{aligned}$$

Step 2: Substitute the expression $\frac{-3}{2}x+\frac{1}{2}$ for y in equation (2).

$$4x-3\left(\tfrac{-3}{2}x+\tfrac{1}{2}\right)=3$$

Step 3: Solve the equation obtained in step 2.

$$\begin{aligned} 4x - 3\left(\frac{-3}{2}x + \frac{1}{2}\right) &= 3 \qquad \text{Multiply both sides by the LCD, } 2. \\ 4x + \frac{9}{2}x - \frac{3}{2} &= 3 \\ 8x + 9x - 3 &= 6 \\ 17x - 3 &= 6 \\ 17x &= 9 \\ x &= \frac{9}{17} \end{aligned}$$

Step 4: Substitute $x = \frac{9}{17}$ into the equation obtained in step 1, $y = \frac{-3}{2}x + \frac{1}{2}$.

$y = \frac{-3}{2}\left(\frac{9}{17}\right) + \frac{1}{2}$

$y = \frac{-27}{34} + \frac{17}{34} = \frac{-10}{34} = \frac{-5}{17}$

We now have $x = \frac{9}{17}$ and $y = \frac{-5}{17}$.

Step 5: Substitution will show that these values of x and y check.

Step 6: The solution is $\left(\frac{9}{17}, \frac{-5}{17}\right)$.

Practice Set D

Exercise:

Problem: Solve the system $\begin{cases} 9x - 5y = -4 \\ 2x + 7y = -9 \end{cases}$

Solution:

These lines intersect at the point $(-1, -1)$.

Exercises

For the following problems, solve the systems by substitution.

Exercise:

Problem: $\begin{cases} 3x + 2y &= 9 \\ y &= -3x + 6 \end{cases}$

Solution:

$(1, 3)$

Exercise:

Problem: $\begin{cases} 5x - 3y & = & -6 \\ \quad y & = & -4x + 12 \end{cases}$

Exercise:

Problem: $\begin{cases} 2x + 2y & = & 0 \\ x & = & 3y - 4 \end{cases}$

Solution:

$(-1, 1)$

Exercise:

Problem: $\begin{cases} 3x + 5y & = & 9 \\ x & = & 4y - 14 \end{cases}$

Exercise:

Problem: $\begin{cases} -3x + y & = & -4 \\ 2x + 3y & = & 10 \end{cases}$

Solution:

$(2, 2)$

Exercise:

Problem: $\begin{cases} -4x + y & = & -7 \\ 2x + 5y & = & 9 \end{cases}$

Exercise:

Problem: $\begin{cases} 6x - 6 & = & 18 \\ x + 3y & = & 3 \end{cases}$

Solution:

$\left(4, -\frac{1}{3}\right)$

Exercise:

Problem: $\begin{cases} -x - y & = & 5 \\ 2x + y & = & 5 \end{cases}$

Exercise:

Problem: $\begin{cases} -5x+y &= 4 \\ 10x-2y &= -8 \end{cases}$

Solution:

Dependent (same line)

Exercise:

Problem: $\begin{cases} x+4y &= 1 \\ -3x-12y &= -1 \end{cases}$

Exercise:

Problem: $\begin{cases} 4x-2y &= 8 \\ 6x+3y &= 0 \end{cases}$

Solution:

$(1,-2)$

Exercise:

Problem: $\begin{cases} 2x+3y &= 12 \\ 2x+4y &= 18 \end{cases}$

Exercise:

Problem: $\begin{cases} 3x-9y &= 6 \\ 6x-18y &= 5 \end{cases}$

Solution:

inconsistent (parallel lines)

Exercise:

Problem: $\begin{cases} -x+4y &= 8 \\ 3x-12y &= 10 \end{cases}$

Exercise:

Problem: $\begin{cases} x+y &= -6 \\ x-y &= 4 \end{cases}$

Solution:

$(-1,-5)$

Exercise:

Problem: $\begin{cases} 2x + y & = & 0 \\ x - 3y & = & 0 \end{cases}$

Exercise:

Problem: $\begin{cases} 4x - 2y & = & 7 \\ \quad y & = & 4 \end{cases}$

Solution:

$\left(\frac{15}{4}, 4\right)$

Exercise:

Problem: $\begin{cases} x + 6y & = & 11 \\ x & = & -1 \end{cases}$

Exercise:

Problem: $\begin{cases} 2x - 4y & = & 10 \\ 3x = 5y & + & 12 \end{cases}$

Solution:

$(-1, -3)$

Exercise:

Problem: $\begin{cases} y + 7x + 4 & = & 0 \\ x = -7y & + & 28 \end{cases}$

Exercise:

Problem: $\begin{cases} x + 4y = 0 \\ x + \frac{2}{3}y = \frac{10}{3} \end{cases}$

Solution:

$(4, -1)$

Exercise:

Problem: $\begin{cases} x = 24 - 5y \\ x - \frac{5}{4}y = \frac{3}{2} \end{cases}$

Exercise:

Problem: $\begin{cases} x = 11 - 6y \\ 3x + 18y = -33 \end{cases}$

Solution:

inconsistent (parallel lines)

Exercise:

Problem: $\begin{cases} 2x + \frac{1}{3}y = 4 \\ 3x + 6y = 39 \end{cases}$

Exercise:

Problem: $\begin{cases} \frac{4}{5}x + \frac{1}{2}y = \frac{3}{10} \\ \frac{1}{3}x + \frac{1}{2}y = \frac{-1}{6} \end{cases}$

Solution:

$(1, -1)$

Exercise:

Problem: $\begin{cases} x - \frac{1}{3}y & = & \frac{-8}{3} \\ -3x + y & = & 1 \end{cases}$

Exercises For Review

Exercise:

Problem:([link]) Find the quotient: $\frac{x^2-x-12}{x^2-2x-15} \div \frac{x^2-3x-10}{x^2-2x-8}$.

Solution:

$\frac{(x-4)^2}{(x-5)^2}$

Exercise:

Problem:([link]) Find the difference: $\frac{x+2}{x^2+5x+6} - \frac{x+1}{x^2+4x+3}$.

Exercise:

Problem:([link]) Simplify $-\sqrt{81x^8y^5z^4}$.

Solution:

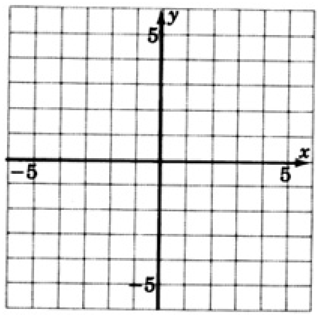
$-9x^4y^2z^2\sqrt{y}$

Exercise:

Problem:(**[link]**) Use the quadratic formula to solve $2x^2 + 2x - 3 = 0.$

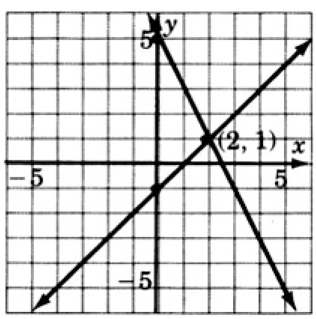
Exercise:

Problem:(**[link]**) Solve by graphing $\begin{cases} x - y = 1 \\ 2x + y = 5 \end{cases}$



Solution:

$(2, 1)$



Introduction to Systems of Linear Equations: Solving by Graphing
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Beginning with the graphical solution of systems, this chapter includes an interpretation of independent, inconsistent, and dependent systems and examples to illustrate the applications for these systems. The substitution method and the addition method of solving a system by elimination are explained, noting when to use each method. The five-step method is again used to illustrate the solutions of value and rate problems (coin and mixture problems), using drawings that correspond to the actual situation. Objectives of this module: be able to recognize a system of equations and a solution to it, be able to graphically interpret independent, inconsistent, and dependent systems, be able to solve a system of linear equations graphically.

Overview

- Systems of Equations
- Solution to A System of Equations
- Graphs of Systems of Equations
- Independent, Inconsistent, and Dependent Systems
- The Method of Solving A System Graphically

Systems of Equations

Systems of Equations
A collection of two linear equations in two variables is called a **system of linear equations in two variables**, or more briefly, **a system of equations**. The **pair** of equations

$$\begin{cases} 5x - 2y = 5 \\ x + y = 8 \end{cases}$$

is a system of equations. The brace $\{$ is used to denote that the two equations occur together (simultaneously).

Solution to A System of Equations

Solution to a System
We know that one of the infinitely many solutions to one linear equation in two variables is an ordered pair. An ordered pair that is a solution to both of the equations in a system is called a **solution to the system of equations**. For example, the ordered pair $(3, 5)$ is a solution to the system

$$\begin{cases} 5x - 2y = 5 \\ x + y = 8 \end{cases}$$

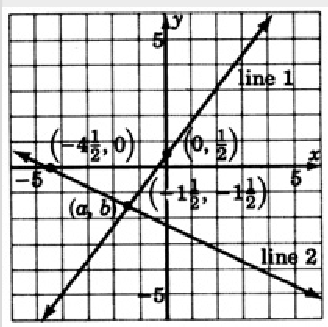
since $(3,\ 5)$ is a solution to both equations.

$$\begin{aligned} 5x - 2y &= 5 \\ 5(3) - 2(5) &= 5 \quad \text{Is this correct?} \\ 15 - 10 &= 5 \quad \text{Is this correct?} \\ 5 &= 5 \quad \text{Yes, this is correct.} \end{aligned} \qquad \begin{aligned} x + y &= 8 \\ 3 + 5 &= 8 \quad \text{Is this correct?} \\ 8 &= 8 \quad \text{Yes, this is correct.} \end{aligned}$$

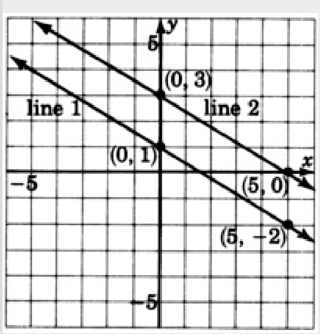
Graphs of Systems of Equations

One method of solving a system of equations is by graphing. We know that the graph of a linear equation in two variables is a straight line. The graph of a system will consist of two straight lines. When two straight lines are graphed, one of three possibilities may result.

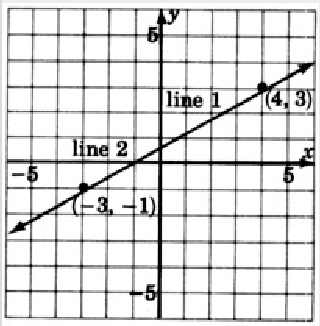
Example:
The lines intersect at the point $(a,\ b)$. The point $(a,\ b)$ is the solution to the corresponding system.



Example:
The lines are parallel. They do not intersect. The system has no solution.



Example:
The lines are coincident (one on the other). They intersect at infinitely many points. The system has infinitely many solutions.



Independent, Inconsistent, and Dependent Systems

Independent Systems
Systems in which the lines intersect at precisely one point are called **independent systems**. In applications, independent systems can arise when the collected data are accurate and complete. For example,

The sum of two numbers is 10 and the product of the two numbers is 21. Find the numbers.

In this application, the data are accurate and complete. The solution is 7 and 3.

Inconsistent Systems
Systems in which the lines are parallel are called **inconsistent systems**. In applications, inconsistent systems can arise when the collected data are contradictory. For example,

The sum of two even numbers is 30 and the difference of the same two numbers is 0. Find the numbers.

The data are contradictory. There is no solution to this application.

Dependent Systems
Systems in which the lines are coincident are called **dependent systems**. In applications, dependent systems can arise when the collected data are incomplete. For example.

The difference of two numbers is 9 and twice one number is 18 more than twice the other.

The data are incomplete. There are infinitely many solutions.

The Method of Solving A System Graphically

The Method of Solving a System Graphically

To solve a system of equations graphically: Graph both equations.

1. If the lines intersect, the solution is the ordered pair that corresponds to the point of intersection. The system is independent.
2. If the lines are parallel, there is no solution. The system is inconsistent.
3. If the lines are coincident, there are infinitely many solutions. The system is dependent.

Sample Set A

Solve each of the following systems by graphing.

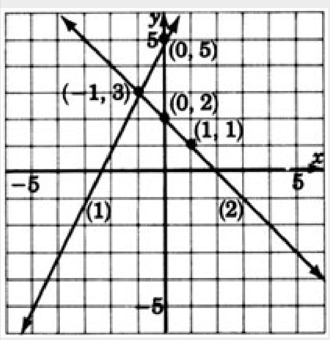
Example:

$$\begin{cases} 2x + y = 5 & (1) \\ x + y = 2 & (2) \end{cases}$$

Write each equation in slope-intercept form.

$$\begin{array}{llll} (1) & -2x + y = 5 & (2) & x + y = 2 \\ & y = 2x + 5 & & y = -x + 2 \end{array}$$

Graph each of these equations.



The lines appear to intersect at the point $(-1,\ 3)$. The solution to this system is $(-1,\ 3)$, or

$x = -1, \quad y = 3$

Check: Substitute $x = -1$, $y = 3$ into each equation.

$$\begin{array}{lrcll} (1) & -2x + y & = & 5 & \\ & -2(-1) + 3 & = & 5 & \text{Is this correct?} \\ & 2 + 3 & = & 5 & \text{Is this correct?} \\ & 5 & = & 5 & \text{Yes, this is correct.} \end{array}$$

$$\begin{array}{lrcll} (2) & x + y & = & 2 & \\ & -1 + 3 & = & 2 & \text{Is this correct?} \\ & 2 & = & 2 & \text{Yes, this is correct.} \end{array}$$

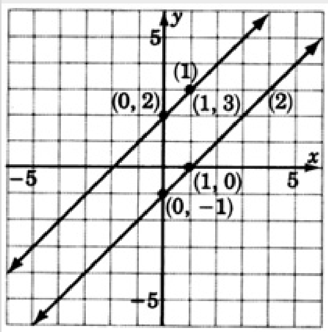
Example:

$$\begin{cases} -x+y=-1 & (1) \\ -x+y=2 & (2) \end{cases}$$

Write each equation in slope-intercept form.

$$\begin{aligned} (1) \qquad -x+y &= -1 \qquad\qquad (2) \qquad -x+y = 2 \\ y &= x-1 \qquad\qquad\qquad\qquad\quad y = x+2 \end{aligned}$$

Graph each of these equations.



These lines are parallel. This system has no solution. We denote this fact by writing **inconsistent**.

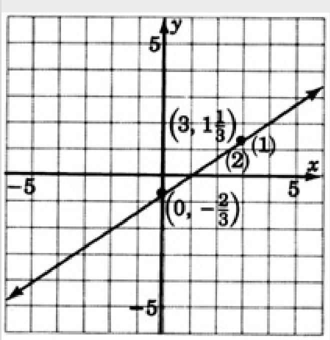
We are sure that these lines are parallel because we notice that they have the same slope, $m=1$ for both lines. The lines are not coincident because the y-intercepts are different.

Example:

$$\begin{cases} -2x+3y=-2 & (1) \\ -6x+9y=-6 & (2) \end{cases}$$

Write each equation in slope-intercept form.

$$\begin{aligned} (1) \qquad -2x+3y &= -2 \qquad\qquad (2) \qquad -6x+9y = -6 \\ 3y &= 2x-2 \qquad\qquad\qquad\qquad\quad 9y = 6x-6 \\ y &= \tfrac{2}{3}x-\tfrac{2}{3} \qquad\qquad\qquad\qquad\quad y = \tfrac{2}{3}x-\tfrac{2}{3} \end{aligned}$$



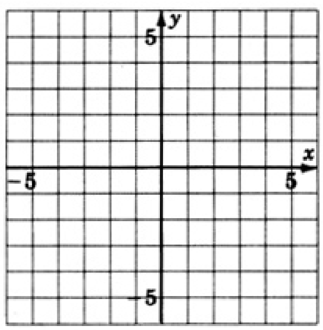
Both equations are the same. This system has infinitely many solutions. We write **dependent**.

Practice Set A

Solve each of the following systems by graphing. Write the ordered pair solution or state that the system is inconsistent, or dependent.

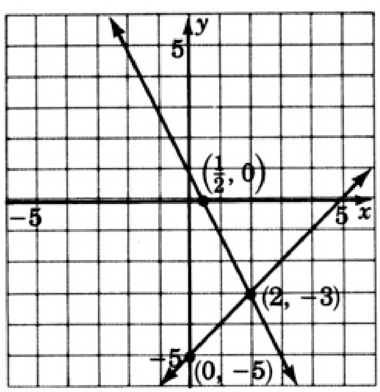
Exercise:

Problem: $\begin{cases} 2x + y = 1 \\ -x + y = -5 \end{cases}$



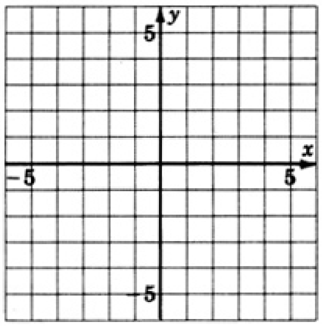
Solution:

$x = 2, y = -3$



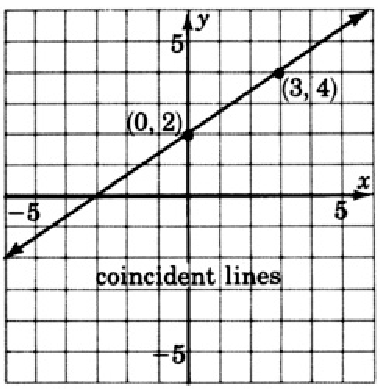
Exercise:

Problem: $\begin{cases} -2x + 3y = 6 \\ 6x - 9y = -18 \end{cases}$



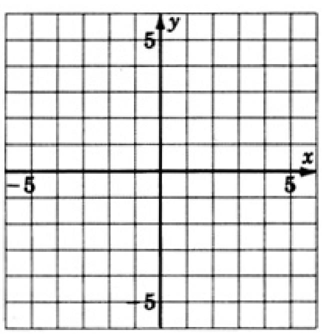
Solution:

dependent



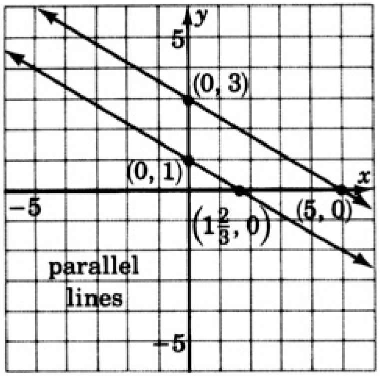
Exercise:

Problem: $\begin{cases} 3x + 5y = 15 \\ 9x + 15y = 15 \end{cases}$



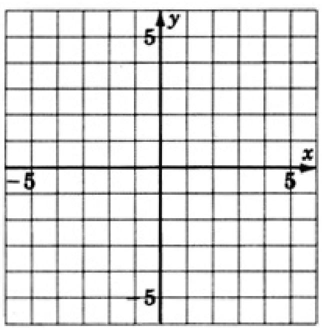
Solution:

inconsistent



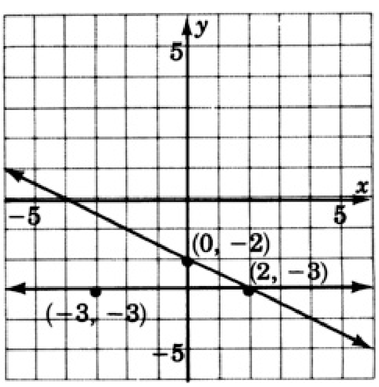
Exercise:

Problem: $\begin{cases} y = -3 \\ x + 2y = -4 \end{cases}$



Solution:

$x = 2, y = -3$

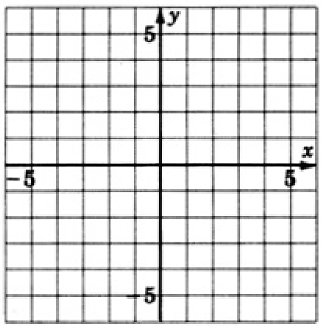


Exercises

For the following problems, solve the systems by graphing. Write the ordered pair solution, or state that the system is inconsistent or dependent.

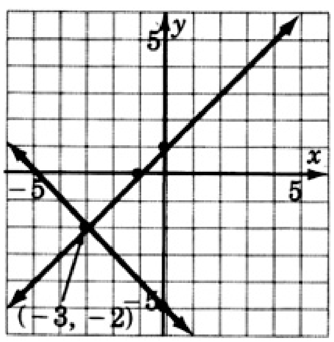
Exercise:

Problem: $\begin{cases} x + y = -5 \\ -x + y = \ \ 1 \end{cases}$



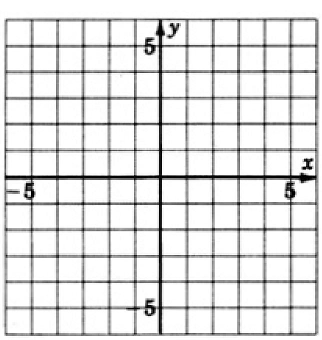
Solution:

$(-3, -2)$



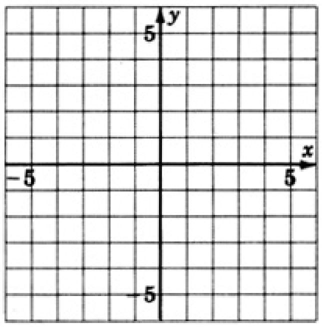
Exercise:

Problem: $\begin{cases} x + y = 4 \\ x + y = 0 \end{cases}$



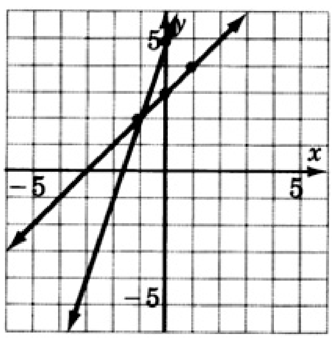
Exercise:

Problem: $\begin{cases} -3x + y = 5 \\ -\ x + y = 3 \end{cases}$



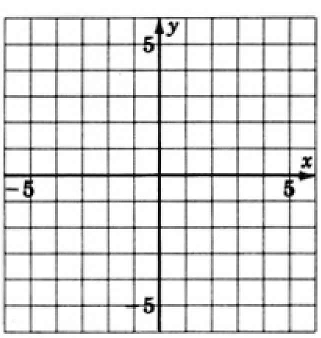
Solution:

$(-1, 2)$



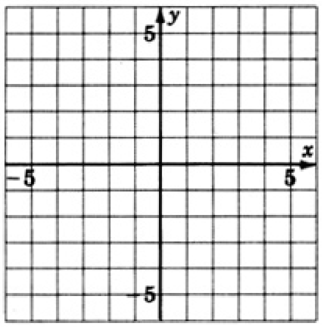
Exercise:

Problem: $\begin{cases} x - y \ = -6 \\ x \ + 2y = \ 0 \end{cases}$



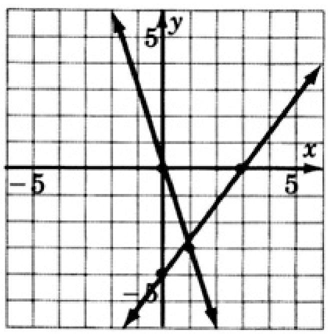
Exercise:

Problem: $\begin{cases} 3x + \quad y = 0 \\ 4x - 3y = \ 12 \end{cases}$



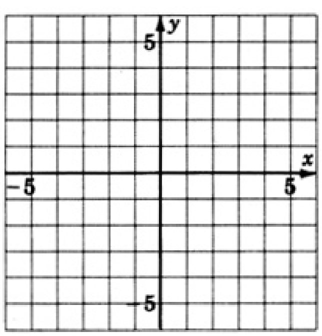
Solution:

$\left(\frac{12}{13}, -\frac{36}{13}\right)$



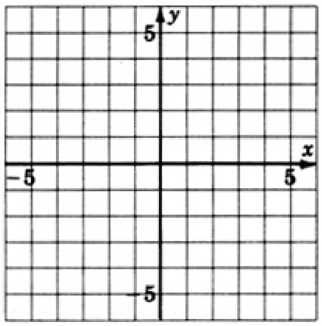
Exercise:

Problem: $\begin{cases} -4x + \ y = 7 \\ -3x + \ y = 2 \end{cases}$



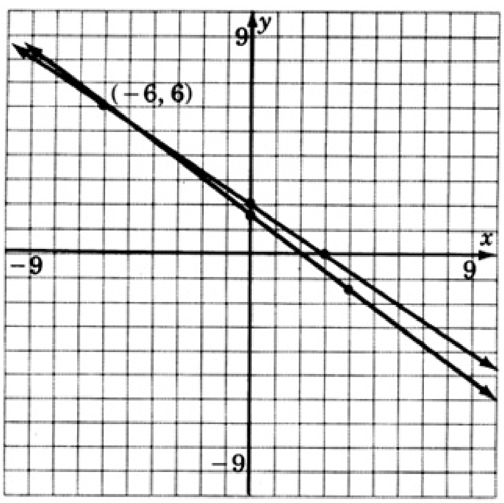
Exercise:

Problem: $\begin{cases} 2x + 3y = \ 6 \\ 3x + 4y = \ 6 \end{cases}$



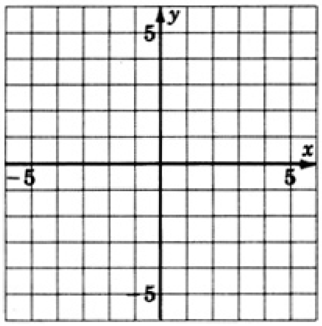
Solution:

These coordinates are hard to estimate. This problem illustrates that the graphical method is not always the most accurate. $(-6, 6)$



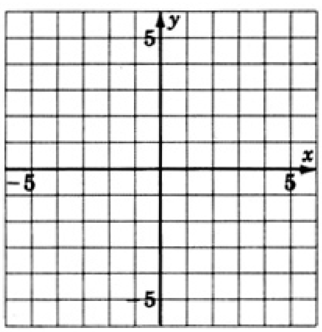
Exercise:

Problem: $\begin{cases} \ x + \ \ y = -3 \\ 4x + \ 4y = -12 \end{cases}$



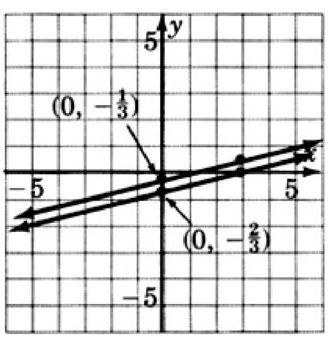
Exercise:

Problem: $\begin{cases} 2x - 3y = 1 \\ 4x - 6y = 4 \end{cases}$



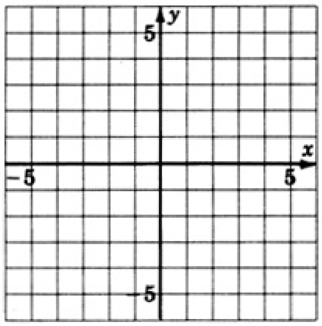
Solution:

inconsistent



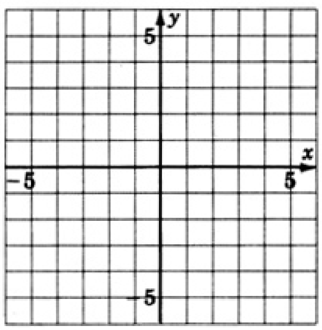
Exercise:

Problem: $\begin{cases} x + 2y = 3 \\ -3x - 6y = -9 \end{cases}$



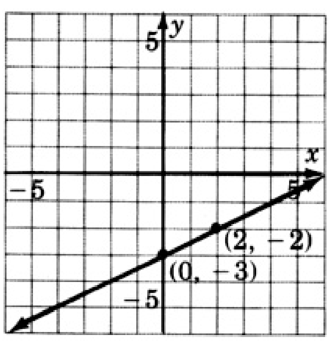
Exercise:

Problem: $\begin{cases} x - 2y = 6 \\ 3x - 6y = 18 \end{cases}$



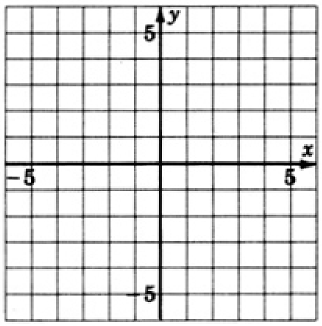
Solution:

dependent



Exercise:

Problem: $\begin{cases} 2x + 3y = 6 \\ -10x - 15y = 30 \end{cases}$



Exercises For Review

Exercise:

Problem:(**[link]**) Express 0.000426 in scientific notation.

Solution:

4.26×10^{-4}

Exercise:

Problem:(**[link]**) Find the product: $(7x-3)^2$.

Exercise:

Problem:

(**[link]**) Supply the missing word. The of a line is a measure of the steepness of the line.

Solution:

slope

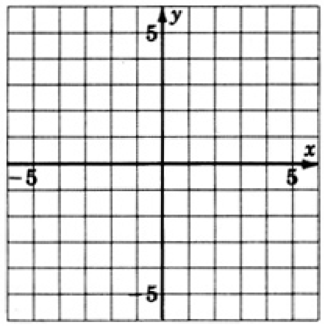
Exercise:

Problem:

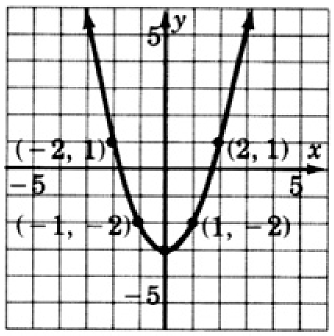
(**[link]**) Supply the missing word. An equation of the form $ax^2+bx+c=0, a \neq 0$, is called a equation.

Exercise:

Problem:(**[link]**) Construct the graph of the quadratic equation $y = x^2 - 3$.



Solution:



Addition and Subtraction of Polynomials
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Operations with algebraic expressions and numerical evaluations are introduced in this chapter. Coefficients are described rather than merely defined. Special binomial products have both literal and symbolic explanations and since they occur so frequently in mathematics, we have been careful to help the student remember them. In each example problem, the student is "talked" through the symbolic form. Objectives of this module: understand the concept of like terms, be able to combine like terms, be able to simplify expressions containing parentheses.

Overview

- Like Terms
- Combining Like Terms
- Simplifying Expressions Containing Parentheses

Like Terms

Like Terms
Terms whose variable parts, including the exponents, are identical are called **like terms**. Like terms is an appropriate name since terms with identical variable parts and different numerical coefficients represent different amounts of the same quantity. As long as we are dealing with quantities of the same type we can combine them using addition and subtraction.

Simplifying an Algebraic Expression
An algebraic expression can be **simplified** by combining like terms.

Sample Set A

Combine the like terms.

Example:

$6 \text{ houses } + \text{ } 4 \text{ houses} = \text{ } 10 \text{ houses}$. 6 and 4 of the same type give 10 of that type.

Example:
$6 \text{ houses} + 4 \text{ houses} + 2 \text{ motels} = 10 \text{ houses} + 2 \text{ motels}$. 6 and 4 of the same type give 10 of that type. Thus, we have 10 of one type and 2 of another type.

Example:
Suppose we let the letter x represent "house." Then, $6x + 4x = 10x$. 6 and 4 of the same type give 10 of that type.

Example:
Suppose we let x represent "house" and y represent "motel."
Equation:

$$6x + 4x + 2y = 10x + 2y$$

Practice Set A

Like terms with the same numerical coefficient represent equal amounts of the same quantity.
Exercise:

Problem: Like terms with different numerical coefficients represent .

Solution:

different amounts of the same quantity

Combining Like Terms

Since like terms represent amounts of the same quantity, they may be combined, that is, like terms may be added together.

Sample Set B

Simplify each of the following polynomials by combining like terms.

Example:
$2x + 5x + 3x$.
There are $2x$'s, then 5 more, then 3 more. This makes a total of $10x$'s.
$2x + 5x + 3x = 10x$

Example:
$7x + 8y - 3x$.
From $7x$'s, we lose $3x$'s. This makes $4x$'s. The $8y$'s represent a quantity different from the x's and therefore will not combine with them.
$7x + 8y - 3x = 4x + 8y$

Example:
$4a^3 - 2a^2 + 8a^3 + a^2 - 2a^3$.
$4a^3$, $8a^3$, and $-2a^3$ represent quantities of the same type.
$4a^3 + 8a^3 - 2a^3 = 10a^3$
$-2a^2$ and a^2 represent quantities of the same type.
$-2a^2 + a^2 = -a^2$
Thus,
$4a^3 - 2a^2 + 8a^3 + a^2 - 2a^3 = 10a^3 - a^2$

Practice Set B

Simplify each of the following expressions.

Exercise:

Problem: $4y + 7y$

Solution:

$11y$

Exercise:

Problem: $3x + 6x + 11x$

Solution:

$20x$

Exercise:

Problem: $5a + 2b + 4a - b - 7b$

Solution:

$9a - 6b$

Exercise:

Problem: $10x^3 - 4x^3 + 3x^2 - 12x^3 + 5x^2 + 2x + x^3 + 8x$

Solution:

$-5x^3 + 8x^2 + 10x$

Exercise:

Problem: $2a^5 - a^5 + 1 - 4ab - 9 + 9ab - 2 - 3 - a^5$

Solution:

$5ab - 13$

Simplifying Expressions Containing Parentheses

Simplifying Expressions Containing Parentheses
When parentheses occur in expressions, they must be removed before the expression can be simplified. Parentheses can be removed using the distributive property.

Distributive Property

$$a(b + c) = ab + ac$$

Sample Set C

Simplify each of the following expressions by using the distributive property and combining like terms.

Example:

$6a + 5(a + 3)$	**Multiply.**
$6a + 5a + 15$	**Combine the like terms $6a$ and $5a$.**
$11a + 15$	

Example:

$4x + 9(x^2 - 6x - 2) + 5$	Remove parentheses.
$4x + 9x^2 - 54x - 18 + 5$	Combine like terms.
$-50x + 9x^2 - 13$	

By convention, the terms in an expression are placed in descending order with the highest degree term appearing first. Numerical terms are placed at the right end of the expression. The commutative property of addition allows us to change the order of the terms.
$9x^2 - 50x - 13$

Example:
$2 + 2[5 + 4(1 + a)]$
Eliminate the innermost set of parentheses first.
$2 + 2[5 + 4 + 4a]$
By the order of operations, simplify inside the parentheses before multiplying (by the 2).

$$\begin{array}{ll} 2 + 2[9 + 4a] & \text{Remove this set of parentheses.} \\ 2 + 18 + 8a & \text{Combine like terms.} \\ 20 + 8a & \text{Write in descending order.} \\ 8a + 20 & \end{array}$$

Example:
$x(x - 3) + 6x(2x + 3)$
Use the rule for multiplying powers with the same base.

$$\begin{array}{ll} x^2 - 3x + 12x^2 + 18x & \text{Combine like terms.} \\ 13x^2 + 15x & \end{array}$$

Practice Set C

Simplify each of the following expressions by using the distributive property and combining like terms.
Exercise:

Problem: $4(x + 6) + 3(2 + x + 3x^2) - 2x^2$

Solution:

$7x^2 + 7x + 30$

Exercise:

Problem: $7(x + x^3) - 4x^3 - x + 1 + 4(x^2 - 2x^3 + 7)$

Solution:

$-5x^3 + 4x^2 + 6x + 29$

Exercise:

Problem: $5(a + 2) + 6a - 7 + (8 + 4)\,(a + 3a + 2)$

Solution:

$59a + 27$

Exercise:

Problem: $x(x + 3) + 4x^2 + 2x$

Solution:

$5x^2 + 5x$

Exercise:

Problem: $a^3(a^2 + a + 5) + a(a^4 + 3a^2 + 4) + 1$

Solution:

$2a^5 + a^4 + 8a^3 + 4a + 1$

Exercise:

Problem: $2[8-3(x-3)]$

Solution:

$-6x+34$

Exercise:

Problem: $x^2+3x+7[x+4x^2+3(x+x^2)]$

Solution:

$50x^2+31x$

Exercises

For the following problems, simplify each of the algebraic expressions.

Exercise:

Problem: $x+3x$

Solution:

$4x$

Exercise:

Problem: $4x+7x$

Exercise:

Problem: $9a+12a$

Solution:

$21a$

Exercise:

Problem: $5m-3m$

Exercise:

Problem: $10x-7x$

Solution:

$3x$

Exercise:

Problem: $7y-9y$

Exercise:

Problem: $6k-11k$

Solution:

$-5k$

Exercise:

Problem: $3a+5a+2a$

Exercise:

Problem: $9y+10y+2y$

Solution:

$21y$

Exercise:

Problem: $5m-7m-2m$

Exercise:

Problem: $h - 3h - 5h$

Solution:

$-7h$

Exercise:

Problem: $a + 8a + 3a$

Exercise:

Problem: $7ab + 4ab$

Solution:

$11ab$

Exercise:

Problem: $8ax + 2ax + 6ax$

Exercise:

Problem: $3a^2 + 6a^2 + 2a^2$

Solution:

$11a^2$

Exercise:

Problem: $14a^2b + 4a^2b + 19a^2b$

Exercise:

Problem: $10y - 15y$

Solution:

$-5y$

Exercise:

Problem: $7ab - 9ab + 4ab$

Exercise:

Problem:

$$210ab^4 + 412ab^4 + 100a^4b \quad \text{(Look closely at the exponents.)}$$

Solution:

$622ab^4 + 100a^4b$

Exercise:

Problem:

$$5x^2y^0 + 3x^2y + 2x^2y + 1, \quad y \neq 0 \quad \text{(Look closely at the exponents.)}$$

Exercise:

Problem: $8w^2 - 12w^2 - 3w^2$

Solution:

$-7w^2$

Exercise:

Problem: $6xy - 3xy + 7xy - 18xy$

Exercise:

Problem: $7x^3 - 2x^2 - 10x + 1 - 5x^2 - 3x^3 - 12 + x$

Solution:

$4x^3 - 7x^2 - 9x - 11$

Exercise:

Problem: $21y - 15x + 40xy - 6 - 11y + 7 - 12x - xy$

Exercise:

Problem: $1x + 1y - 1x - 1y + x - y$

Solution:

$x - y$

Exercise:

Problem: $5x^2 - 3x - 7 + 2x^2 - x$

Exercise:

Problem: $-2z^3 + 15z + 4z^3 + z^2 - 6z^2 + z$

Solution:

$2z^3 - 5z^2 + 16z$

Exercise:

Problem: $18x^2y - 14x^2y - 20x^2y$

Exercise:

Problem: $-9w^5 - 9w^4 - 9w^5 + 10w^4$

Solution:

$-18w^5 + w^4$

Exercise:

Problem: $2x^4 + 4x^3 - 8x^2 + 12x - 1 - 7x^3 - 1x^4 - 6x + 2$

Exercise:

Problem: $17d^3r + 3d^3r - 5d^3r + 6d^2r + d^3r - 30d^2r + 3 - 7 + 2$

Solution:

$16d^3r - 24d^2r - 2$

Exercise:

Problem: $a^0 + 2a^0 - 4a^0, \quad a \neq 0$

Exercise:

Problem: $4x^0 + 3x^0 - 5x^0 + 7x^0 - x^0, \quad x \neq 0$

Solution:

8

Exercise:

Problem: $2a^3b^2c + 3a^2b^2c^0 + 4a^2b^2 - a^3b^2c, \quad c \neq 0$

Exercise:

Problem: $3z - 6z + 8z$

Solution:

$5z$

Exercise:

Problem: $3z^2 - z + 3z^3$

Exercise:

Problem: $6x^3 + 12x + 5$

Solution:

$6x^3 + 12x + 5$

Exercise:

Problem: $3(x+5) + 2x$

Exercise:

Problem: $7(a+2) + 4$

Solution:

$7a + 18$

Exercise:

Problem: $y + 5(y+6)$

Exercise:

Problem: $2b + 6(3-5b)$

Solution:

$-28b + 18$

Exercise:

Problem: $5a - 7c + 3(a - c)$

Exercise:

Problem: $8x - 3x + 4(2x + 5) + 3(6x - 4)$

Solution:

$31x + 8$

Exercise:

Problem: $2z + 4ab + 5z - ab + 12(1 - ab - z)$

Exercise:

Problem: $(a + 5)4 + 6a - 20$

Solution:

$10a$

Exercise:

Problem: $(4a + 5b - 2)3 + 3(4a + 5b - 2)$

Exercise:

Problem: $(10x + 3y^2)4 + 4(10x + 3y^2)$

Solution:

$80x + 24y^2$

Exercise:

Problem: $2(x - 6) + 5$

Exercise:

Problem: $1(3x+15)+2x-12$

Solution:

$5x+3$

Exercise:

Problem: $1(2+9a+4a^2)+a^2-11a$

Exercise:

Problem: $1(2x-6b+6a^2b+8b^2)+1(5x+2b-3a^2b)$

Solution:

$3a^2b+8b^2-4b+7x$

Exercise:

Problem:

After observing the following problems, can you make a conjecture about $1(a+b)$?
$1(a+b)\ =$

Exercise:

Problem: Using the result of problem 52, is it correct to write $(a+b)=a+b$?

Solution:

yes

Exercise:

Problem: $3(2a + 2a^2) + 8(3a + 3a^2)$

Exercise:

Problem: $x(x + 2) + 2(x^2 + 3x - 4)$

Solution:

$3x^2 + 8x - 8$

Exercise:

Problem: $A(A + 7) + 4(A^2 + 3a + 1)$

Exercise:

Problem: $b(2b^3 + 5b^2 + b + 6) - 6b^2 - 4b + 2$

Solution:

$2b^4 + 5b^3 - 5b^2 + 2b + 2$

Exercise:

Problem: $4a - a(a + 5)$

Exercise:

Problem: $x - 3x(x^2 - 7x - 1)$

Solution:

$-3x^3 + 21x^2 + 4x$

Exercise:

Problem: $ab(a - 5) - 4a^2b + 2ab - 2$

Exercise:

Problem: $xy(3xy+2x-5y)-2x^2y^2-5x^2y+4xy^2$

Solution:

$x^2y^2-3x^2y-xy^2$

Exercise:

Problem: $3h[2h+5(h+2)]$

Exercise:

Problem: $2k[5k+3(1+7k)]$

Solution:

$52k^2+6k$

Exercise:

Problem: $8a[2a-4ab+9(a-5-ab)]$

Exercise:

Problem: $6\{m+5n[n+3(n-1)]+2n^2\}-4n^2-9m$

Solution:

$128n^2-90n-3m$

Exercise:

Problem: $5[4(r-2s)-3r-5s]+12s$

Exercise:

Problem: $8\{9[b-2a+6c(c+4)-4c^2]+4a+b\}-3b$

Solution:

$144c^2-112a+77b+1728c$

Exercise:

Problem: $5[4(6x-3)+x]-2x-25x+4$

Exercise:

Problem: $3xy^2(4xy+5y)+2xy^3+6x^2y^3+4y^3-12xy^3$

Solution:

$18x^2y^3+5xy^3+4y^3$

Exercise:

Problem:

$9a^3b^7(a^3b^5-2a^2b^2+6)-2a(a^2b^7-5a^5b^{12}+3a^4b^9)-a^3b^7$

Exercise:

Problem: $-8(3a+2)$

Solution:

$-24a-16$

Exercise:

Problem: $-4(2x-3y)$

Exercise:

Problem: $-4xy^2[7xy - 6(5 - xy^2) + 3(-xy + 1) + 1]$

Solution:

$-24x^2y^4 - 16x^2y^3 + 104xy^2$

Exercises for Review

Exercise:

Problem: ([link]) Simplify $\left(\frac{x^{10}y^8z^2}{x^2y^6}\right)^3$.

Exercise:

Problem: ([link]) Find the value of $\frac{-3(4-9)-6(-3)-1}{2^3}$.

Solution:

4

Exercise:

Problem:

([link]) Write the expression $\frac{42x^2y^5z^3}{21x^4y^7}$ so that no denominator appears.

Exercise:

Problem: ([link]) How many $(2a+5)$'s are there in $3x(2a+5)$?

Solution:

$3x$

Exercise:

Problem: ([link]) Simplify $3(5n + 6m^2) - 2(3n + 4m^2)$.

Basic Properties of Exponents
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The symbols, notations, and properties of numbers that form the basis of algebra, as well as exponents and the rules of exponents, are introduced in this chapter. Each property of real numbers and the rules of exponents are expressed both symbolically and literally. Literal explanations are included because symbolic explanations alone may be difficult for a student to interpret. Objectives of this module: understand exponential notation, be able to read exponential notation, understand how to use exponential notation with the order of operations.

Overview

- Exponential Notation
- Reading Exponential Notation
- The Order of Operations

Exponential Notation

In Section [link] we were reminded that multiplication is a description for repeated addition. A natural question is "Is there a description for **repeated** multiplication?" The answer is yes. The notation that describes repeated multiplication is **exponential notation**.

Factors
In multiplication, the numbers being multiplied together are called **factors**. In repeated multiplication, all the factors are the same. In nonrepeated multiplication, none of the factors are the same. For example,

Example:

$18 \cdot 18 \cdot 18 \cdot 18$	Repeated multiplication of 18. All four factors, 18, are the same.
$x \cdot x \cdot x \cdot x \cdot x$	Repeated multiplication of x. All five factors, x, are the same.
$3 \cdot 7 \cdot a$	Nonrepeated multiplication. None of the factors are the same.

Exponential notation is used to show repeated multiplication of the same factor. The notation consists of using a **superscript on the factor that is repeated**. The superscript is called an **exponent**.

Exponential Notation

If x is any real number and n is a natural number, then

$$x^n = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors of } x}$$

An exponent records the number of identical factors in a multiplication.

Note that the definition for exponential notation only has meaning for natural number exponents. We will extend this notation to include other numbers as exponents later.

Sample Set A

Example:
$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^6$.
The repeated factor is 7. The exponent 6 records the fact that 7 appears 6 times in the multiplication.

Example:
$x \cdot x \cdot x \cdot x = x^4$.
The repeated factor is x. The exponent 4 records the fact that x appears 4 times in the multiplication.

Example:
$(2y)(2y)(2y) = (2y)^3$.
The repeated factor is $2y$. The exponent 3 records the fact that the factor $2y$ appears 3 times in the multiplication.

Example:
$2yyy = 2y^3$.
The repeated factor is y. The exponent 3 records the fact that the factor y appears 3 times in the multiplication.

Example:

$(a+b)(a+b)(a-b)(a-b)(a-b) = (a+b)^2(a-b)^3$.
The repeated factors are $(a+b)$ and $(a-b)$, $(a+b)$ appearing 2 times and $(a-b)$ appearing 3 times.

Practice Set A

Write each of the following using exponents.
Exercise:

Problem: $a \cdot a \cdot a \cdot a$

Solution:

a^4

Exercise:

Problem: $(3b)(3b)(5c)(5c)(5c)(5c)$

Solution:

$(3b)^2(5c)^4$

Exercise:

Problem: $2 \cdot 2 \cdot 7 \cdot 7 \cdot 7 \cdot (a-4)(a-4)$

Solution:

$2^2 \cdot 7^3(a-4)^2$

Exercise:

Problem: $8xxxyzzzzz$

Solution:

$8x^3yz^5$

CAUTION

It is extremely important to realize and remember that an exponent applies only to the factor to which it is directly connected.

Sample Set B

Example:
$8x^3$ means $8 \cdot xxx$ and **not** $8x8x8x$. The exponent 3 applies only to the factor x since it is only to the factor x that the 3 is connected.

Example:
$(8x)^3$ means $(8x)(8x)(8x)$ since the parentheses indicate that the exponent 3 is directly connected to the factor $8x$. Remember that the grouping symbols $(\quad)$ indicate that the quantities inside are to be considered as one single number.

Example:
$34(a+1)^2$ means $34 \cdot (a+1)(a+1)$ since the exponent 2 applies only to the factor $(a+1)$.

Practice Set B

Write each of the following without exponents.
Exercise:

Problem: $4a^3$

Solution:

$4aaa$

Exercise:

Problem: $(4a)^3$

Solution:

$(4a)(4a)(4a)$

Sample Set C

Example:
Select a number to show that $(2x)^2$ is not always equal to $2x^2$.
Suppose we choose x to be 5. Consider both $(2x)^2$ and $2x^2$.
Equation:

$$\begin{array}{lcl} (2x)^2 & & 2x^2 \\ (2 \cdot 5)^2 & & 2 \cdot 5^2 \\ (10)^2 & & 2 \cdot 25 \\ 100 & \neq & 50 \end{array}$$

Notice that $(2x)^2 = 2x^2$ only when $x = 0$.

Practice Set C

Exercise:

Problem: Select a number to show that $(5x)^2$ is not always equal to $5x^2$.

Solution:

Select $x = 3$. Then $(5 \cdot 3)^2 = (15)^2 = 225$, but $5 \cdot 3^2 = 5 \cdot 9 = 45$. $\quad 225 \neq 45$.

Reading Exponential Notation

In x^n,

Base
x is the **base**

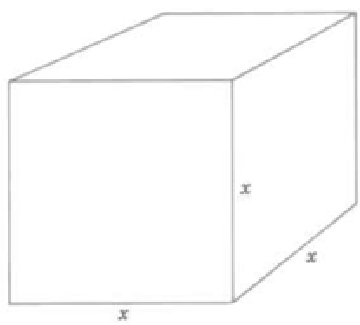
Exponent
n is the **exponent**

Power
The number represented by x^n is called a **power**.

x to the nth Power
The term x^n is read as "x to the n th power," or more simply as "x to the nth."

x Squared and x Cubed
The symbol x^2 is often read as "x squared," and x^3 is often read as "x cubed." A natural question is "Why are geometric terms appearing in the exponent expression?" The answer for x^3 is this: x^3 means $x \cdot x \cdot x$. In geometry, the volume of a rectangular box is found by multiplying the length by the width by the depth. A cube has the same length on each side. If we represent this length by the letter x then the volume of the cube is $x \cdot x \cdot x$, which, of course, is described by x^3. (Can you think of why x^2 is read as x squared?)

Cube with
length $= x$
width $= x$
depth $= x$
Volume $= xxx = x^3$



The Order of Operations

In Section [link] we were introduced to the order of operations. It was noted that we would insert another operation before multiplication and division. We can do that now.
The Order of Operations

1. Perform all operations inside grouping symbols beginning with the innermost set.
2. Perform all exponential **operations as you come to** them, moving left-to-right.
3. Perform all multiplications and divisions as you come to them, moving left-to-right.
4. Perform all additions and subtractions as you come to them, moving left-to-right.

Sample Set D

Use the order of operations to simplify each of the following.

Example:
$2^2 + 5 = 4 + 5 = 9$

Example:
$5^2 + 3^2 + 10 = 25 + 9 + 10 = 44$

Example:
$$\begin{aligned} 2^2 + (5)(8) - 1 &= 4 + (5)(8) - 1 \\ &= 4 + 40 - 1 \\ &= 43 \end{aligned}$$

Example:
$$\begin{aligned} 7 \cdot 6 - 4^2 + 1^5 &= 7 \cdot 6 - 16 + 1 \\ &= 42 - 16 + 1 \\ &= 27 \end{aligned}$$

Example:
$$\begin{aligned} (2+3)^3 + 7^2 - 3(4+1)^2 &= (5)^3 + 7^2 - 3(5)^2 \\ &= 125 + 49 - 3(25) \\ &= 125 + 49 - 75 \\ &= 99 \end{aligned}$$

Example:

$$\begin{aligned} \left[4(6+2)^3\right]^2 &= \left[4(8)^3\right]^2 \\ &= \left[4(512)\right]^2 \\ &= \left[2048\right]^2 \\ &= 4,194,304 \end{aligned}$$

Example:

$$\begin{aligned} 6(3^2+2^2)+4^2 &= 6(9+4)+4^2 \\ &= 6(13)+4^2 \\ &= 6(13)+16 \\ &= 78+16 \\ &= 94 \end{aligned}$$

Example:

$$\begin{aligned} \frac{6^2+2^2}{4^2+6\cdot 2^2}+\frac{1^3+8^2}{10^2-(19)(5)} &= \frac{36+4}{16+6\cdot 4}+\frac{1+64}{100-95} \\ &= \frac{36+4}{16+24}+\frac{1+64}{100-95} \\ &= \frac{40}{40}+\frac{65}{5} \\ &= 1+13 \\ &= 14 \end{aligned}$$

Practice Set D

Use the order of operations to simplify the following.

Exercise:

Problem: $3^2+4\cdot 5$

Solution:

29

Exercise:

Problem: $2^3+3^3-8\cdot 4$

Solution:

3

Exercise:

Problem: $1^4 + (2^2 + 4)^2 \div 2^3$

Solution:

9

Exercise:

Problem: $[6(10 - 2^3)]^2 - 10^2 - 6^2$

Solution:

8

Exercise:

Problem: $\frac{5^2+6^2-10}{1+4^2} + \frac{0^4-0^5}{7^2-6\cdot 2^3}$

Solution:

3

Exercises

For the following problems, write each of the quantities using exponential notation.

Exercise:

Problem: b to the fourth

Solution:

b^4

Exercise:

Problem: a squared

Exercise:

Problem: x to the eighth

Solution:

x^8

Exercise:

Problem: (-3) cubed

Exercise:

Problem: 5 times s squared

Solution:

$5s^2$

Exercise:

Problem: 3 squared times y to the fifth

Exercise:

Problem: a cubed minus $(b+7)$ squared

Solution:

$a^3-(b+7)^2$

Exercise:

Problem: $(21-x)$ cubed plus $(x+5)$ to the seventh

Exercise:

Problem: $xxxxx$

Solution:

x^5

Exercise:

Problem: $(8)(8)xxxx$

Exercise:

Problem: $2 \cdot 3 \cdot 3 \cdot 3 \cdot 3xxyyyyy$

Solution:

$2\left(3^4\right)x^2y^5$

Exercise:

Problem: $2 \cdot 2 \cdot 5 \cdot 6 \cdot 6 \cdot 6xyyzzzwwww$

Exercise:

Problem: $7xx(a+8)(a+8)$

Solution:

$7x^2(a+8)^2$

Exercise:

Problem: $10xyy(c+5)(c+5)(c+5)$

Exercise:

Problem: $4x4x4x4x4x$

Solution:

$(4x)^5$ or 4^5x^5

Exercise:

Problem: $(9a)(9a)(9a)(9a)$

Exercise:

Problem: $(-7)(-7)(-7)aabbba(-7)baab$

Solution:

$(-7)^4 a^5 b^5$

Exercise:

Problem: $(a-10)(a-10)(a+10)$

Exercise:

Problem: $(z+w)(z+w)(z+w)(z-w)(z-w)$

Solution:

$(z+w)^3 (z-w)^2$

Exercise:

Problem: $(2y)(2y)2y2y$

Exercise:

Problem: $3xyxxy-(x+1)(x+1)(x+1)$

Solution:

$3x^3y^2-(x+1)^3$

For the following problems, expand the quantities so that no exponents appear.

Exercise:

Problem: 4^3

Exercise:

Problem: 6^2

Solution:

$6 \cdot 6$

Exercise:

Problem: $7^3 y^2$

Exercise:

Problem: $8x^3 y^2$

Solution:

$8 \cdot x \cdot x \cdot x \cdot y \cdot y$

Exercise:

Problem: $(18x^2 y^4)^2$

Exercise:

Problem: $(9a^3 b^2)^3$

Solution:

$(9aaabb)\,(9aaabb)\,(9aaabb)$ or $9 \ \cdot \ 9 \ \cdot \ 9aaaaaaaaabbbbbb$

Exercise:

Problem: $5x^2(2y^3)^3$

Exercise:

Problem:$10a^3 b^2(3c)^2$

Solution:

$10aaabb\,(3c)\,(3c)$ or $10 \ \cdot \ 3 \ \cdot \ 3aaabbcc$

Exercise:

Problem: $(a+10)^2(a^2+10)^2$

Exercise:

Problem: $(x^2-y^2)(x^2+y^2)$

Solution:

$(xx - yy)(xx + yy)$

For the following problems, select a number (or numbers) to show that
Exercise:

Problem: $(5x)^2$ is not generally equal to $5x^2$.

Exercise:

Problem: $(7x)^2$ is not generally equal to $7x^2$.

Solution:

Select $x = 2$. Then, $196 \neq 28$.

Exercise:

Problem: $(a + b)^2$ is not generally equal to $a^2 + b^2$.

Exercise:

Problem: For what real number is $(6a)^2$ equal to $6a^2$?

Solution:

zero

Exercise:

Problem: For what real numbers, a and b, is $(a + b)^2$ equal to $a^2 + b^2$?

Use the order of operations to simplify the quantities for the following problems.
Exercise:

Problem: $3^2 + 7$

Solution:

16

Exercise:

Problem: $4^3 - 18$

Exercise:

Problem: $5^2 + 2(40)$

Solution:

105

Exercise:

Problem: $8^2 + 3 + 5(2 + 7)$

Exercise:

Problem: $2^5 + 3(8 + 1)$

Solution:

59

Exercise:

Problem: $3^4 + 2^4(1 + 5)^3$

Exercise:

Problem: $(6^2 - 4^2) \div 5$

Solution:

4

Exercise:

Problem: $2^2(10 - 2^3)$

Exercise:

Problem: $(3^4 - 4^3) \div 17$

Solution:

1

Exercise:

Problem: $(4+3)^2 + 1 \div (2 \cdot 5)$

Exercise:

Problem: $\left(2^4 + 2^5 - 2^3 \cdot 5\right)^2 \div 4^2$

Solution:

4

Exercise:

Problem: $1^6 + 0^8 + 5^2(2+8)^3$

Exercise:

Problem: $(7)(16) - 9^2 + 4(1^1 + 3^2)$

Solution:

71

Exercise:

Problem: $\frac{2^3 - 7}{5^2}$

Exercise:

Problem: $\frac{(1+6)^2 + 2}{19}$

Solution:

$\frac{51}{19}$

Exercise:

Problem: $\frac{6^2-1}{5}+\frac{4^3+(2)(3)}{10}$

Exercise:

Problem: $\frac{5[8^2-9(6)]}{2^5-7}+\frac{7^2-4^2}{2^4-5}$

Solution:

5

Exercise:

Problem: $\frac{(2+1)^3+2^3+1^3}{6^2}-\frac{15^2-[2(5)]^2}{5\cdot 5^2}$

Exercise:

Problem: $\frac{6^3-2\cdot 10^2}{2^2}+\frac{18(2^3+7^2)}{2(19)-3^3}$

Solution:

$\frac{1070}{11}$ or 97.27

Exercises for Review

Exercise:

Problem:

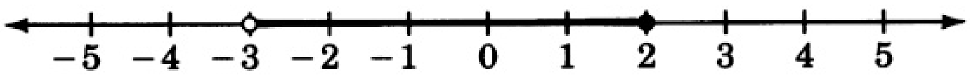
(**[link]**) Use algebraic notation to write the statement "a number divided by eight, plus five, is equal to ten."

Exercise:

Problem:

(**[link]**) Draw a number line that extends from -5 to 5 and place points at all real numbers that are strictly greater than -3 but less than or equal to 2.

Solution:



Exercise:

Problem: ([link]) Is every integer a whole number?

Exercise:

Problem:

([link]) Use the commutative property of multiplication to write a number equal to the number yx.

Solution:

xy

Exercise:

Problem: ([link]) Use the distributive property to expand $3(x+6)$.

Exponent Power Rules
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The symbols, notations, and properties of numbers that form the basis of algebra, as well as exponents and the rules of exponents, are introduced in this chapter. Each property of real numbers and the rules of exponents are expressed both symbolically and literally. Literal explanations are included because symbolic explanations alone may be difficult for a student to interpret. Objectives of this module: understand the power rules for powers, products, and quotients.

Overview

- The Power Rule for Powers
- The Power Rule for Products
- The Power Rule for quotients

The Power Rule for Powers

The following examples suggest a rule for raising a power to a power:

Example:
$(a^2)^3 = a^2 \cdot a^2 \cdot a^2$
Using the product rule we get
$(a^2)^3 = a^{2+2+2}$

$(a^2)^3 = a^{3 \cdot 2}$

$(a^2)^3 = a^6$

Example:

$$\begin{aligned} (x^9)^4 &= x^9 \cdot x^9 \cdot x^9 \cdot x^9 \\ (x^9)^4 &= x^{9+9+9+9} \\ (x^9)^4 &= x^{4 \cdot 9} \\ (x^9)^4 &= x^{36} \end{aligned}$$

POWER RULE FOR POWERS
If x is a real number and n and m are natural numbers,
$(x^n)^m = x^{n \cdot m}$

To raise a power to a power, multiply the exponents.

Sample Set A

Simplify each expression using the power rule for powers. All exponents are natural numbers.

Example:
$(x^3)^4 = \boxed{x^{3 \cdot 4}}\, x^{12}$ The box represents a step done mentally.

Example:

$$(y^5)^3 = \boxed{y^{5 \cdot 3}} = y^{15}$$

Example:

$$(d^{20})^6 = \boxed{d^{20 \cdot 6}} = d^{120}$$

Example:
$(x^{\square})^{\triangle} = x^{\square\triangle}$

Although we don’t know exactly what number $\square\triangle$ is, the notation $\square\triangle$ indicates the multiplication.

Practice Set A

Simplify each expression using the power rule for powers.
Exercise:

Problem: $(x^5)^4$

Solution:

x^{20}

Exercise:

Problem: $\left(y^7\right)^7$

Solution:

y^{49}

The Power Rule for Products

The following examples suggest a rule for raising a product to a power:

Example:

$$\begin{aligned}(ab)^3 &= ab \cdot ab \cdot ab \quad \text{Use the commutative property of multiplication.} \\ &= aaabbb \\ &= a^3b^3\end{aligned}$$

Example:

$$\begin{aligned}(xy)^5 &= xy \cdot xy \cdot xy \cdot xy \cdot xy \\ &= xxxxx \cdot yyyyy \\ &= x^5y^5\end{aligned}$$

Example:

$$\begin{aligned}(4xy\text{z})^2 &= 4xyz \cdot 4xyz \\ &= 4 \cdot 4 \cdot xx \cdot yy \cdot zz \\ &= 16x^2y^2z^2\end{aligned}$$

POWER RULE FOR PRODUCTS
If x and y are real numbers and n is a natural number,
$(xy)^n = x^ny^n$

To raise a product to a power, apply the exponent to each and every factor.

Sample Set B

Make use of either or both the power rule for products and power rule for powers to simplify each expression.

Example:
$(ab)^7 = a^7b^7$

Example:
$(axy)^4 = a^4x^4y^4$

Example:
$(3ab)^2 = 3^2a^2b^2 = 9a^2b^2 \quad \text{Don't forget to apply the exponent to the 3!}$

Example:
$(2st)^5 = 2^5s^5t^5 = 32s^5t^5$

Example:
$(ab^3)^2 = a^2(b^3)^2 = a^2b^6$ We used two rules here. First, the power rule for products. Second, the power rule for powers.

Example:
$$\begin{aligned}(7a^4b^2c^8)^2 &= 7^2(a^4)^2(b^2)^2(c^8)^2\\ &= 49a^8b^4c^{16}\end{aligned}$$

Example:
If $6a^3c^7 \neq 0$, then $(6a^3c^7)^0 = 1$ Recall that $x^0 = 1$ for $x \neq 0$.

Example:
$$\begin{aligned}\left[2(x+1)^4\right]^6 &= 2^6(x+1)^{24}\\ &= 64(x+1)^{24}\end{aligned}$$

Practice Set B

Make use of either or both the power rule for products and the power rule for powers to simplify each expression.

Exercise:

Problem: $(ax)^4$

Solution:

a^4x^4

Exercise:

Problem: $(3bxy)^2$

Solution:

$9b^2x^2y^2$

Exercise:

Problem: $[4t\,(s-5)]^3$

Solution:

$64t^3(s-5)^3$

Exercise:

Problem: $(9x^3y^5)^2$

Solution:

$81x^6y^{10}$

Exercise:

Problem: $(1a^5b^8c^3d)^6$

Solution:

$a^{30}b^{48}c^{18}d^6$

Exercise:

Problem: $[(a+8)\,(a+5)]^4$

Solution:

$(a+8)^4(a+5)^4$

Exercise:

Problem: $\left[(12c^4u^3(w-3)^2\right]^5$

Solution:

$12^5c^{20}u^{15}(w-3)^{10}$

Exercise:

Problem: $\left[10t^4y^7j^3d^2v^6n^4g^8(2-k)^{17}\right]^4$

Solution:

$10^4t^{16}y^{28}j^{12}d^8v^{24}n^{16}g^{32}(2-k)^{68}$

Exercise:

Problem: $(x^3x^5y^2y^6)^9$

Solution:

$\left(x^8y^8\right)^9=x^{72}y^{72}$

Exercise:

Problem: $(10^6\cdot 10^{12}\cdot 10^5)^{10}$

Solution:

10^{230}

The Power Rule for Quotients

The following example suggests a rule for raising a quotient to a power.

Example:
$\left(\frac{a}{b}\right)^3=\frac{a}{b}\cdot\frac{a}{b}\cdot\frac{a}{b}=\frac{a\cdot a\cdot a}{b\cdot b\cdot b}=\frac{a^3}{b^3}$

POWER RULE FOR QUOTIENTS
If x and y are real numbers and n is a natural number,
$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n},\ y \neq 0$

To raise a quotient to a power, distribute the exponent to both the numerator and denominator.

Sample Set C

Make use of the power rule for quotients, the power rule for products, the power rule for powers, or a combination of these rules to simplify each expression. All exponents are natural numbers.

Example:
$\left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$

Example:
$\left(\frac{a}{c}\right)^2 = \frac{a^2}{c^2}$

Example:
$\left(\frac{2x}{b}\right)^4 = \frac{(2x)^4}{b^4} = \frac{2^4x^4}{b^4} = \frac{16x^4}{b^4}$

Example:
$\left(\frac{a^3}{b^5}\right)^7 = \frac{(a^3)^7}{(b^5)^7} = \frac{a^{21}}{b^{35}}$

Example:
$\left(\frac{3c^4r^2}{2^3g^5}\right)^3 = \frac{3^3c^{12}r^6}{2^9g^{15}} = \frac{27c^{12}r^6}{2^9g^{15}}$ or $\frac{27c^{12}r^6}{512g^{15}}$

Example:
$\left[\frac{(a-2)}{(a+7)}\right]^4 = \frac{(a-2)^4}{(a+7)^4}$

Example:

$$\left[\frac{6x(4-x)^4}{2a(y-4)^6}\right]^2 = \frac{6^2x^2(4-x)^8}{2^2a^2(y-4)^{12}} = \frac{36x^2(4-x)^8}{4a^2(y-4)^{12}} = \frac{9x^2(4-x)^8}{a^2(y-4)^{12}}$$

Example:

$$\left(\frac{a^3b^5}{a^2b}\right)^3 = (a^{3-2}b^{5-1})^3$$

We can simplify within the parentheses. We have a rule that tells us to proceed this way.

$$= (ab^4)^3$$

$$= a^3b^{12}$$

$$\left(\frac{a^3b^5}{a^2b}\right)^3 = \frac{a^9b^{15}}{a^6b^3} = a^{9-6}b^{15-3} = a^3b^{12}$$

We could have actually used the power rule for quotients first. Distribute the exponent, then simplify using the other rules.

It is probably better, for the sake of consistency, to work inside the parentheses first.

Example:

$$\left(\frac{a^rb^s}{c^t}\right)^w = \frac{a^{rw}b^{sw}}{c^{tw}}$$

Practice Set C

Make use of the power rule for quotients, the power rule for products, the power rule for powers, or a combination of these rules to simplify each expression.

Exercise:

Problem: $\left(\frac{a}{c}\right)^5$

Solution:

$\frac{a^5}{c^5}$

Exercise:

Problem: $\left(\frac{2x}{3y}\right)^3$

Solution:

$\frac{8x^3}{27y^3}$

Exercise:

Problem: $\left(\frac{x^2y^4z^7}{a^5b}\right)^9$

Solution:

$\frac{x^{18}y^{36}z^{63}}{a^{45}b^9}$

Exercise:

Problem: $\left[\frac{2a^4(b-1)}{3b^3(c+6)}\right]^4$

Solution:

$\frac{16a^{16}(b-1)^4}{81b^{12}(c+6)^4}$

Exercise:

Problem: $\left(\frac{8a^3b^2c^6}{4a^2b}\right)^3$

Solution:

$8a^3b^3c^{18}$

Exercise:

Problem: $\left[\frac{(9+w)^2}{(3+w)^5}\right]^{10}$

Solution:

$\frac{(9+w)^{20}}{(3+w)^{50}}$

Exercise:

Problem: $\left[\frac{5x^4(y+1)}{5x^4(y+1)}\right]^6$

Solution:

$1, \text{ if } x^4(y+1) \neq 0$

Exercise:

Problem: $\left(\frac{16x^3v^4c^7}{12x^2vc^6}\right)^0$

Solution:

$1, \text{ if } x^2vc^6 \neq 0$

Exercises

Use the power rules for exponents to simplify the following problems. Assume that all bases are nonzero and that all variable exponents are natural numbers.

Exercise:

Problem: $(ac)^5$

Solution:

a^5c^5

Exercise:

Problem: $(nm)^7$

Exercise:

Problem: $(2a)^3$

Solution:

$8a^3$

Exercise:

Problem: $(2a)^5$

Exercise:

Problem: $(3xy)^4$

Solution:

$81x^4y^4$

Exercise:

Problem: $(2xy)^5$

Exercise:

Problem: $(3ab)^4$

Solution:

$81a^4b^4$

Exercise:

Problem: $(6mn)^2$

Exercise:

Problem: $(7y^3)^2$

Solution:

$49y^6$

Exercise:

Problem: $(3m^3)^4$

Exercise:

Problem: $(5x^6)^3$

Solution:

$125x^{18}$

Exercise:

Problem: $(5x^2)^3$

Exercise:

Problem: $(10a^2b)^2$

Solution:

$100a^4b^2$

Exercise:

Problem: $(8x^2y^3)^2$

Exercise:

Problem: $(x^2y^3z^5)^4$

Solution:

$x^8y^{12}z^{20}$

Exercise:

Problem: $(2a^5b^{11})^0$

Exercise:

Problem: $(x^3y^2z^4)^5$

Solution:

$x^{15}y^{10}z^{20}$

Exercise:

Problem: $(m^6n^2p^5)^5$

Exercise:

Problem: $(a^4b^7c^6d^8)^8$

Solution:

$a^{32}b^{56}c^{48}d^{64}$

Exercise:

Problem: $(x^2y^3z^9w^7)^3$

Exercise:

Problem: $(9xy^3)^0$

Solution:

1

Exercise:

Problem: $\left(\frac{1}{2}f^2r^6s^5\right)^4$

Exercise:

Problem: $\left(\frac{1}{8}c^{10}d^8e^4f^9\right)^2$

Solution:

$\frac{1}{64}c^{20}d^{16}e^8f^{18}$

Exercise:

Problem: $\left(\frac{3}{5}a^3b^5c^{10}\right)^3$

Exercise:

Problem: $(xy)^4(x^2y^4)$

Solution:

x^6y^8

Exercise:

Problem: $(2a^2)^4(3a^5)^2$

Exercise:

Problem: $(a^2b^3)^3(a^3b^3)^4$

Solution:

$a^{18}b^{21}$

Exercise:

Problem: $(h^3k^5)^2(h^2k^4)^3$

Exercise:

Problem: $(x^4y^3z)^4(x^5yz^2)^2$

Solution:

$x^{26}y^{14}z^{8}$

Exercise:

Problem: $(ab^3c^2)^5(a^2b^2c)^2$

Exercise:

Problem: $\frac{(6a^2b^8)^2}{(3ab^5)^2}$

Solution:

$4a^2b^6$

Exercise:

Problem: $\frac{(a^3b^4)^5}{(a^4b^4)^3}$

Exercise:

Problem: $\frac{(x^6y^5)^3}{(x^2y^3)^5}$

Solution:

x^8

Exercise:

Problem: $\frac{(a^8b^{10})^3}{(a^7b^5)^3}$

Exercise:

Problem: $\frac{(m^5n^6p^4)^4}{(m^4n^5p)^4}$

Solution:

$m^4n^4p^{12}$

Exercise:

Problem: $\frac{(x^8y^3z^2)^5}{(x^6yz)^6}$

Exercise:

Problem: $\frac{\left(10x^4y^5z^{11}\right)^3}{\left(xy^2\right)^4}$

Solution:

$1000x^8y^7z^{33}$

Exercise:

Problem: $\frac{(9a^4b^5)(2b^2c)}{(3a^3b)(6bc)}$

Exercise:

Problem: $\frac{\left(2x^3y^3\right)^4\left(5x^6y^8\right)^2}{\left(4x^5y^3\right)^2}$

Solution:

$25x^{14}y^{22}$

Exercise:

Problem: $\left(\frac{3x}{5y}\right)^2$

Exercise:

Problem: $\left(\frac{3ab}{4xy}\right)^3$

Solution:

$\frac{27a^3b^3}{64x^3y^3}$

Exercise:

Problem: $\left(\frac{x^2y^2}{2z^3}\right)^5$

Exercise:

Problem: $\left(\frac{3a^2b^3}{c^4}\right)^3$

Solution:

$\frac{27a^6b^9}{c^{12}}$

Exercise:

Problem: $\left(\frac{4^2a^3b^7}{b^5c^4}\right)^2$

Exercise:

Problem: $\left[\frac{x^2(y-1)^3}{(x+6)}\right]^4$

Solution:

$\frac{x^8(y-1)^{12}}{(x+6)^4}$

Exercise:

Problem: $\left(x^n t^{2m}\right)^4$

Exercise:

Problem: $\frac{\left(x^{n+2}\right)^3}{x^{2n}}$

Solution:

x^{n+6}

Exercise:

Problem: $(xy)^{\triangle}$

Exercise:

Problem:

$(2ab)^{\star}$

Solution:

$2^{\star}a^{\star}b^{\star}$

Exercise:

Problem:

$\frac{(3a^{\triangle}b^{\nabla})^{\square}}{(5xy^{\diamond})^{\star}}$

Exercise:

Problem:

$$\frac{10m^{\Delta}}{5m^{\star}}$$

Solution:

$$2m^{\Delta-\star}$$

Exercise:

Problem: $\frac{4^3a^{\Delta}a^{\square}}{4a^{\nabla}}$

Exercise:

Problem: $\left(\frac{4x^{\Delta}}{2y^{\nabla}}\right)^{\square}$

Solution:

$$\frac{2^{\square}x^{\Delta\square}}{y^{\nabla\square}}$$

Exercise:

Problem:

$$\left(\frac{16a^3b^{\star}}{5a^{\Delta}b^{\nabla}}\right)^0$$

Exercises for Review

Exercise:

Problem: ([link]) Is there a smallest integer? If so, what is it?

Solution:

no

Exercise:

Problem: ([link]) Use the distributive property to expand $5a\left(2x+8\right)$.

Exercise:

Problem: ([link]) Find the value of $\frac{\left(5-3\right)^{2}+\left(5+4\right)^{3}+2}{4^{2}-2\cdot 5-1}$.

Solution:

147

Exercise:

Problem: ([link]) Assuming the bases are not zero, find the value of $\left(4a^{2}b^{3}\right)\left(5ab^{4}\right)$.

Exercise:

Problem: ([link]) Assuming the bases are not zero, find the value of $\frac{36x^{10}y^{8}z^{3}w^{0}}{9x^{5}y^{2}z}$.

Solution:

$4x^{5}y^{6}z^{2}$

Multiplication of Polynomials
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Operations with algebraic expressions and numerical evaluations are introduced in this chapter. Coefficients are described rather than merely defined. Special binomial products have both literal and symbolic explanations and since they occur so frequently in mathematics, we have been careful to help the student remember them. In each example problem, the student is "talked" through the symbolic form. Objectives of this module: be able to multiply a polynomial by a monomial, be able to simplify +(a + b) and -(a - b), be able to multiply a polynomial by a polynomial.

Overview

- Multiplying a Polynomial by a Monomial
- Simplifying $+(a+b)$ and $-(a+b)$
- Multiplying a Polynomial by a Polynomial

Multiplying a Polynomial by a Monomial

Multiplying a polynomial by a monomial is a direct application of the distributive property.

Distributive Property

$$a(b+c) = ab + ac$$

The distributive property suggests the following rule.

Multiplying a Polynomial by a Monomial
To multiply a polynomial by a monomial, multiply **every** term of the polynomial by the monomial and then add the resulting products together.

Sample Set A

Example:

$$\begin{aligned} 3(x+9) &= 3 \cdot x + 3 \cdot 9 \\ &= 3x + 27 \end{aligned}$$

Example:

$$\begin{aligned} 6(x^3 - 2x) = 6(x^3 + (-2x)) &= 6 \cdot x^3 + 6(-2x) \\ &= 6x^3 - 12x \end{aligned}$$

Example:

$$\begin{aligned}(x-7)x &= x \cdot x + x(-7) \\ &= x^2 - 7x\end{aligned}$$

Example:

$$\begin{aligned}8a^2(3a^4 - 5a^3 + a) &= 8a^2 \cdot 3a^4 + 8a^2(-5a^3) + 8a^2 \cdot a \\ &= 24a^6 - 40a^5 + 8a^3\end{aligned}$$

Example:

$$\begin{aligned}4x^2y^7z(x^5y + 8y^2z^2) &= 4x^2y^7z \cdot x^5y + 4x^2y^7z \cdot 8y^2z^2 \\ &= 4x^7y^8z + 32x^2y^9z^3\end{aligned}$$

Example:
$10ab^2c(125a^2) = 1250a^3b^2c$

Example:

$$\begin{aligned}(9x^2z + 4w)(5zw^3) &= 9x^2z \cdot 5zw^3 + 4w \cdot 5zw^3 \\ &= 45x^2z^2w^3 + 20zw^4 \\ &= 45x^2w^3z^2 + 20w^4z\end{aligned}$$

Practice Set A

Determine the following products.
Exercise:

Problem: $3(x+8)$

Solution:

$3x + 24$
Exercise:

Problem: $(2+a)4$

Solution:

$4a + 8$

Exercise:

Problem: $(a^2 - 2b + 6)2a$

Solution:

$2a^3 - 4ab + 12a$

Exercise:

Problem: $8a^2b^3(2a + 7b + 3)$

Solution:

$16a^3b^3 + 56a^2b^4 + 24a^2b^3$

Exercise:

Problem: $4x(2x^5 + 6x^4 - 8x^3 - x^2 + 9x - 11)$

Solution:

$8x^6 + 24x^5 - 32x^4 - 4x^3 + 36x^2 - 44x$

Exercise:

Problem: $(3a^2b)(2ab^2 + 4b^3)$

Solution:

$6a^3b^3 + 12a^2b^4$

Exercise:

Problem: $5mn(m^2n^2 + m + n^0), \;\; n \neq 0$

Solution:

$5m^3n^3 + 5m^2n + 5mn$

Exercise:

Problem: $6.03(2.11a^3 + 8.00a^2b)$

Solution:

$12.7233a^3 + 48.24a^2b$

Simplifying $+(a+b)$ and $-(a+b)$

$+(a+b)$ **and** $-(a+b)$
Oftentimes, we will encounter multiplications of the form

$+1(a+b)\text{or}-1(a+b)$

These terms will actually appear as

$+(a+b)$ and $-(a+b)$

Using the distributive property, we can remove the parentheses.

$$
\begin{aligned}
+(a+b) = +1(a+b) &= (+1)(a) + (+1)(b) \\
&= a + b
\end{aligned}
$$

The parentheses have been removed and the sign of each term has remained the same.

$$
\begin{aligned}
-(a+b) = -1(a+b) &= (-1)(a) + (-1)(b) \\
&= -a - b
\end{aligned}
$$

The parentheses have been removed and the sign of each term has been changed to its opposite.

1. To remove a set of parentheses preceded by a "$+$" sign, simply remove the parentheses and leave the sign of each term the same.
2. To remove a set of parentheses preceded by a "$-$" sign, remove the parentheses and change the sign of each term to its opposite sign.

Sample Set B

Simplify the expressions.

Example:
$(6x-1)$.

This set of parentheses is preceded by a "$+$" sign (implied). We simply drop the parentheses.
$(6x-1) = 6x-1$

Example:
$(14a^2b^3 - 6a^3b^2 + ab^4) = 14a^2b^3 - 6a^3b^2 + ab^4$

Example:
$-(21a^2 + 7a - 18)$.

This set of parentheses is preceded by a "$-$" sign. We can drop the parentheses as long as we change the sign of **every** term inside the parentheses to its opposite sign.

$$-(21a^2 + 7a - 18) = -21a^2 - 7a + 18$$

Example:
$-(7y^3 - 2y^2 + 9y + 1) = -7y^3 + 2y^2 - 9y - 1$

Practice Set B

Simplify by removing the parentheses.
Exercise:

Problem: $(2a + 3b)$

Solution:

$2a + 3b$

Exercise:

Problem: $(a^2 - 6a + 10)$

Solution:

$a^2 - 6a + 10$

Exercise:

Problem: $-(x + 2y)$

Solution:

$-x - 2y$

Exercise:

Problem: $-(5m - 2n)$

Solution:

$-5m + 2n$

Exercise:

Problem: $-(-3s^2 - 7s + 9)$

Solution:

$3s^2 + 7s - 9$

Multiplying a Polynomial by a Polynomial

Since we can consider an expression enclosed within parentheses as a single quantity, we have, by the distributive property,

$$\begin{aligned}(a+b)(c+d) &= (a+b)c + (a+b)d \\ &= ac + bc + ad + bd\end{aligned}$$

For convenience we will use the commutative property of addition to write this expression so that the first two terms contain a and the second two contain b.

$(a + b)(c + d) = ac + ad + bc + bd$

This method is commonly called the **FOIL method**.

- **F** First terms
- **O** Outer terms
- **I** Inner terms
- **L** Last terms

$$(a+b)(2+3) = \underbrace{(a+b) + (a+b)}_{\text{2 terms}} + \underbrace{(a+b) + (a+b) + (a+b)}_{\text{3 terms}}$$

Rearranging,

$$\begin{aligned}&= a + a + b + b + a + a + a + b + b + b \\ &= 2a + 2b + 3a + 3b\end{aligned}$$

Combining like terms,

$$= 5a + 5b$$

This use of the distributive property suggests the following rule.

Multiplying a Polynomial by a Polynomial
To multiply two polynomials together, multiply **every** term of one polynomial by **every** term of the other polynomial.

Sample Set C

Perform the following multiplications and simplify.

Example:

$$\begin{aligned}(a+6)(a+3) &= a\cdot a + a\cdot 3 + 6\cdot a + 6\cdot 3\\ &= a^2+3a+6a+18\\ &= a^2+9a+18\end{aligned}$$

F: $a\cdot a$
O: $3\cdot a$
I: $6\cdot a$
L: $6\cdot 3$

With some practice, the second and third terms can be combined mentally.

Example:

$$\begin{aligned}(x+y)(2x+4y) &= x\cdot 2x + x\cdot 4y + y\cdot 2x + y\cdot 4y\\ &= 2x^2+4xy+2xy+4y^2\\ &= 2x^2+6xy+4y^2\end{aligned}$$

F: $x\cdot 2x$
O: $x\cdot 4y$
I: $y\cdot 2x$
L: $y\cdot 4y$

Example:

$$\begin{aligned}(x^2+4)(x^2+7x+2) &= x^2\cdot x^2 + x^2\cdot 7x + x^2\cdot 2 + 4\cdot x^2 + 4\cdot 7x + 4\cdot 2\\ &= x^4+7x^3+2x^2+4x^2+28x+8\\ &= x^4+7x^3+6x^2+28x+8\end{aligned}$$

Example:

$$\begin{aligned}(a-4)(a-3) &= a\cdot a + a(-3) - 4\cdot a - 4(-3)\\ &= a^2-3a-4a+12\\ &= a^2-7a+12\end{aligned}$$

F: $a\cdot a$
O: $a\cdot(-3)$
I: $(-4)\cdot a$
L: $(-4)(-3)$

Example:

$$\begin{aligned}(m-3)^2 &= (m-3)(m-3)\\ &= m\cdot m + m(-3) - 3\cdot m - 3(-3)\\ &= m^2-3m-3m+9\\ &= m^2-6m+9\end{aligned}$$

Example:

$$\begin{aligned}(x+5)^3 &= (x+5)(x+5)(x+5) && \text{Associate the first two factors.}\\ &= [(x+5)(x+5)](x+5)\\ &= \left[x^2+5x+5x+25\right](x+5)\\ &= \left[x^2+10x+25\right](x+5)\\ &= x^2\cdot x+x^2\cdot 5+10x\cdot x+10x\cdot 5+25\cdot x+25\cdot 5\\ &= x^3+5x^2+10x^2+50x+25x+125\\ &= x^3+15x^2+75x+125\end{aligned}$$

Practice Set C

Find the following products and simplify.

Exercise:

Problem: $(a+1)(a+4)$

Solution:

a^2+5a+4

Exercise:

Problem: $(m-9)(m-2)$

Solution:

$m^2-11m+18$

Exercise:

Problem: $(2x+4)(x+5)$

Solution:

$2x^2+14x+20$

Exercise:

Problem: $(x+y)(2x-3y)$

Solution:

$2x^2-xy-3y^2$

Exercise:

Problem: $(3a^2 - 1)(5a^2 + a)$

Solution:

$15a^4 + 3a^3 - 5a^2 - a$

Exercise:

Problem: $(2x^2y^3 + xy^2)(5x^3y^2 + x^2y)$

Solution:

$10x^5y^5 + 7x^4y^4 + x^3y^3$

Exercise:

Problem: $(a + 3)(a^2 + 3a + 6)$

Solution:

$a^3 + 6a^2 + 15a + 18$

Exercise:

Problem: $(a + 4)(a + 4)$

Solution:

$a^2 + 8a + 16$

Exercise:

Problem: $(r - 7)(r - 7)$

Solution:

$r^2 - 14r + 49$

Exercise:

Problem: $(x + 6)^2$

Solution:

$x^2 + 12x + 36$

Exercise:

Problem: $(y - 8)^2$

Solution:

$y^2 - 16y + 64$

Sample Set D

Perform the following additions and subtractions.

Example:

$3x + 7 + (x - 3)$.	We must first remove the parentheses. They are preceded by a "+" sign, so we remove them and leave the sign of each term the same.
$3x + 7 + x - 3$	Combine like terms.
$4x + 4$	

Example:

$5y^3 + 11 - (12y^3 - 2)$.	We first remove the parentheses. They are preceded by a "-" sign, so we remove them and change the sign of each term inside them.
$5y^3 + 11 - 12y^3 + 2$	Combine like terms.
$-7y^3 + 13$	

Example:

Add $4x^2 + 2x - 8$ to $3x^2 - 7x - 10$.

$$(4x^2 + 2x - 8) + (3x^2 - 7x - 10)$$
$$4x^2 + 2x - 8 + 3x^2 - 7x - 10$$
$$7x^2 - 5x - 18$$

Example:

Subtract $8x^2 - 5x + 2$ from $3x^2 + x - 12$.

$$(3x^2 + x - 12) - (8x^2 - 5x + 2)$$
$$3x^2 + x - 12 - 8x^2 + 5x - 2$$
$$-5x^2 + 6x - 14$$

Be very careful **not** to write this problem as

$$3x^2 + x - 12 - 8x^2 - 5x + 2$$

This form has us subtracting only the very first term, $8x^2$, rather than the entire expression. Use parentheses.
Another incorrect form is

$$8x^2 - 5x + 2 - (3x^2 + x - 12)$$

This form has us performing the subtraction in the wrong order.

Practice Set D

Perform the following additions and subtractions.
Exercise:

Problem: $6y^2 + 2y - 1 + (5y^2 - 18)$

Solution:

$11y^2 + 2y - 19$

Exercise:

Problem: $(9m - n) - (10m + 12n)$

Solution:

$-m - 13n$

Exercise:

Problem: Add $2r^2 + 4r - 1$ to $3r^2 - r - 7$.

Solution:

$5r^2 + 3r - 8$

Exercise:

Problem: Subtract $4s - 3$ from $7s + 8$.

Solution:

$3s + 11$

Exercises

For the following problems, perform the multiplications and combine any like terms.
Exercise:

Problem: $7(x + 6)$

Solution:

$7x + 42$

Exercise:

Problem: $4(y + 3)$

Exercise:

Problem: $6(y + 4)$

Solution:

$6y + 24$

Exercise:

Problem: $8(m + 7)$

Exercise:

Problem: $5(a - 6)$

Solution:

$5a - 30$

Exercise:

Problem: $2(x - 10)$

Exercise:

Problem: $3(4x + 2)$

Solution:

$12x + 6$

Exercise:

Problem: $6(3x + 4)$

Exercise:

Problem: $9(4y - 3)$

Solution:

$36y - 27$

Exercise:

Problem: $5(8m-6)$

Exercise:

Problem: $-9(a+7)$

Solution:

$-9a-63$

Exercise:

Problem: $-3(b+8)$

Exercise:

Problem: $-4(x+2)$

Solution:

$-4x-8$

Exercise:

Problem: $-6(y+7)$

Exercise:

Problem: $-3(a-6)$

Solution:

$-3a+18$

Exercise:

Problem: $-9(k-7)$

Exercise:

Problem: $-5(2a+1)$

Solution:

$-10a-5$

Exercise:

Problem: $-7(4x+2)$

Exercise:

Problem: $-3(10y-6)$

Solution:

$-30y+18$

Exercise:

Problem: $-8(4y-11)$

Exercise:

Problem: $x(x+6)$

Solution:

x^2+6x

Exercise:

Problem: $y(y+7)$

Exercise:

Problem: $m(m-4)$

Solution:

m^2-4m

Exercise:

Problem: $k(k-11)$

Exercise:

Problem: $3x(x+2)$

Solution:

$3x^2+6x$

Exercise:

Problem: $4y(y+7)$

Exercise:

Problem: $6a(a-5)$

Solution:

$6a^2 - 30a$

Exercise:

Problem: $9x(x-3)$

Exercise:

Problem: $3x(5x+4)$

Solution:

$15x^2 + 12x$

Exercise:

Problem: $4m(2m+7)$

Exercise:

Problem: $2b(b-1)$

Solution:

$2b^2 - 2b$

Exercise:

Problem: $7a(a-4)$

Exercise:

Problem: $3x^2(5x^2+4)$

Solution:

$15x^4 + 12x^2$

Exercise:

Problem: $9y^3(3y^2+2)$

Exercise:

Problem: $4a^4(5a^3+3a^2+2a)$

Solution:

$20a^7 + 12a^6 + 8a^5$

Exercise:

Problem: $2x^4(6x^3-5x^2-2x+3)$

Exercise:

Problem: $-5x^2(x+2)$

Solution:

$-5x^3-10x^2$

Exercise:

Problem: $-6y^3(y+5)$

Exercise:

Problem: $2x^2y(3x^2y^2-6x)$

Solution:

$6x^4y^3-12x^3y$

Exercise:

Problem: $8a^3b^2c(2ab^3+3b)$

Exercise:

Problem: $b^5x^2(2bx-11)$

Solution:

$2b^6x^3-11b^5x^2$

Exercise:

Problem: $4x(3x^2-6x+10)$

Exercise:

Problem: $9y^3(2y^4-3y^3+8y^2+y-6)$

Solution:

$18y^7-27y^6+72y^5+9y^4-54y^3$

Exercise:

Problem: $-a^2b^3(6ab^4+5ab^3-8b^2+7b-2)$

Exercise:

Problem: $(a+4)(a+2)$

Solution:

$a^2 + 6a + 8$

Exercise:

Problem: $(x + 1)(x + 7)$

Exercise:

Problem: $(y + 6)(y - 3)$

Solution:

$y^2 + 3y - 18$

Exercise:

Problem: $(t + 8)(t - 2)$

Exercise:

Problem: $(i - 3)(i + 5)$

Solution:

$i^2 + 2i - 15$

Exercise:

Problem: $(x - y)(2x + y)$

Exercise:

Problem: $(3a - 1)(2a - 6)$

Solution:

$6a^2 - 20a + 6$

Exercise:

Problem: $(5a - 2)(6a - 8)$

Exercise:

Problem: $(6y + 11)(3y + 10)$

Solution:

$18y^2 + 93y + 110$

Exercise:

Problem: $(2t + 6)(3t + 4)$

Exercise:

Problem: $(4 + x)(3 - x)$

Solution:

$-x^2 - x + 12$

Exercise:

Problem: $(6 + a)(4 + a)$

Exercise:

Problem: $(x^2 + 2)(x + 1)$

Solution:

$x^3 + x^2 + 2x + 2$

Exercise:

Problem: $(x^2 + 5)(x + 4)$

Exercise:

Problem: $(3x^2 - 5)(2x^2 + 1)$

Solution:

$6x^4 - 7x^2 - 5$

Exercise:

Problem: $(4a^2b^3 - 2a)(5a^2b - 3b)$

Exercise:

Problem: $(6x^3y^4 + 6x)(2x^2y^3 + 5y)$

Solution:

$12x^5y^7 + 30x^3y^5 + 12x^3y^3 + 30xy$

Exercise:

Problem: $5(x - 7)(x - 3)$

Exercise:

Problem: $4(a+1)(a-8)$

Solution:

$4a^2-28a-32$

Exercise:

Problem: $a(a-3)(a+5)$

Exercise:

Problem: $x(x+1)(x+4)$

Solution:

x^3+5x^2+4x

Exercise:

Problem: $x^2(x+5)(x+7)$

Exercise:

Problem: $y^3(y-3)(y-2)$

Solution:

$y^5-5y^4+6y^3$

Exercise:

Problem: $2a^2(a+4)(a+3)$

Exercise:

Problem: $5y^6(y+7)(y+1)$

Solution:

$5y^8+40y^7+35y^6$

Exercise:

Problem: $ab^2(a^2-2b)(a+b^4)$

Exercise:

Problem: $x^3y^2(5x^2y^2-3)(2xy-1)$

Solution:

$10x^6y^5 - 5x^5y^4 - 6x^4y^3 + 3x^3y^2$

Exercise:

Problem: $6(a^2 + 5a + 3)$

Exercise:

Problem: $8(c^3 + 5c + 11)$

Solution:

$8c^3 + 40c + 88$

Exercise:

Problem: $3a^2(2a^3 - 10a^2 - 4a + 9)$

Exercise:

Problem: $6a^3b^3(4a^2b^6 + 7ab^8 + 2b^{10} + 14)$

Solution:

$24a^5b^9 + 42a^4b^{11} + 12a^3b^{13} + 18a^3b^3$

Exercise:

Problem: $(a - 4)(a^2 + a - 5)$

Exercise:

Problem: $(x - 7)(x^2 + x - 3)$

Solution:

$x^3 - 6x^2 - 10x + 21$

Exercise:

Problem: $(2x + 1)(5x^3 + 6x^2 + 8)$

Exercise:

Problem: $(7a^2 + 2)(3a^5 - 4a^3 - a - 1)$

Solution:

$21a^7 - 22a^5 - 15a^3 - 7a^2 - 2a - 2$

Exercise:

Problem: $(x + y)(2x^2 + 3xy + 5y^2)$

Exercise:

Problem: $(2a+b)(5a^2+4a^2b-b-4)$

Solution:

$10a^3+8a^3b+4a^2b^2+5a^2b-b^2-8a-4b-2ab$

Exercise:

Problem: $(x+3)^2$

Exercise:

Problem: $(x+1)^2$

Solution:

x^2+2x+1

Exercise:

Problem: $(x-5)^2$

Exercise:

Problem: $(a+2)^2$

Solution:

a^2+4a+4

Exercise:

Problem: $(a-9)^2$

Exercise:

Problem: $-(3x-5)^2$

Solution:

$-9x^2+30x-25$

Exercise:

Problem: $-(8t+7)^2$

For the following problems, perform the indicated operations and combine like terms.
Exercise:

Problem: $3x^2 + 5x - 2 + (4x^2 - 10x - 5)$

Solution:

$7x^2 - 5x - 7$

Exercise:

Problem: $-2x^3 + 4x^2 + 5x - 8 + (x^3 - 3x^2 - 11x + 1)$

Exercise:

Problem: $-5x - 12xy + 4y^2 + (-7x + 7xy - 2y^2)$

Solution:

$2y^2 - 5xy - 12x$

Exercise:

Problem: $(6a^2 - 3a + 7) - 4a^2 + 2a - 8$

Exercise:

Problem: $(5x^2 - 24x - 15) + x^2 - 9x + 14$

Solution:

$6x^2 - 33x - 1$

Exercise:

Problem: $(3x^3 - 7x^2 + 2) + (x^3 + 6)$

Exercise:

Problem: $(9a^2b - 3ab + 12ab^2) + ab^2 + 2ab$

Solution:

$9a^2b + 13ab^2 - ab$

Exercise:

Problem: $6x^2 - 12x + (4x^2 - 3x - 1) + 4x^2 - 10x - 4$

Exercise:

Problem: $5a^3 - 2a - 26 + (4a^3 - 11a^2 + 2a) - 7a + 8a^3 + 20$

Solution:

$17a^3 - 11a^2 - 7a - 6$

Exercise:

Problem: $2xy - 15 - (5xy + 4)$

Exercise:

Problem: Add $4x + 6$ to $8x - 15$.

Solution:

$12x - 9$

Exercise:

Problem: Add $5y^2 - 5y + 1$ to $-9y^2 + 4y - 2$.

Exercise:

Problem: Add $3(x + 6)$ to $4(x - 7)$.

Solution:

$7x - 10$

Exercise:

Problem: Add $-2(x^2 - 4)$ to $5(x^2 + 3x - 1)$.

Exercise:

Problem: Add four times $5x + 2$ to three times $2x - 1$.

Solution:

$26x + 5$

Exercise:

Problem: Add five times $-3x + 2$ to seven times $4x + 3$.

Exercise:

Problem: Add -4 times $9x + 6$ to -2 times $-8x - 3$.

Solution:

$-20x - 18$

Exercise:

Problem: Subtract $6x^2 - 10x + 4$ from $3x^2 - 2x + 5$.

Exercise:

Problem: Substract $a^2 - 16$ from $a^2 - 16$.

Solution:

0

Exercises for Review

Exercise:

Problem: ([link]) Simplify $\left(\frac{15x^2y^6}{5xy^2}\right)^4$.

Exercise:

Problem: ([link]) Express the number 198,000 using scientific notation.

Solution:

1.98×10^5

Exercise:

Problem: ([link]) How many $4a^2x^3$'s are there in $-16a^4x^5$?

Exercise:

Problem:

([link]) State the degree of the polynomial $4xy^3 + 3x^5y - 5x^3y^3$, and write the numerical coefficient of each term.

Solution:

$\text{degree is } 6; \ \ 4, 3, -5$

Exercise:

Problem: ([link]) Simplify $3(4x - 5) + 2(5x - 2) - (x - 3)$.

Special Products
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Operations with algebraic expressions and numerical evaluations are introduced in this chapter. Coefficients are described rather than merely defined. Special binomial products have both literal and symbolic explanations and since they occur so frequently in mathematics, we have been careful to help the student remember them. In each example problem, the student is "talked" through the symbolic form. Objectives of this module: be able to expand (a + b)^2, (a - b)^2, and (a + b)(a - b).

Overview

- Expanding $(a+b)^2$ and $(a-b)^2$
- Expanding $(a+b)\,(a-b)$

Three binomial products occur so frequently in algebra that we designate them as **special binomial products**. We have seen them before (Sections [link] and [link]), but we will study them again because of their importance as time saving devices and in solving equations (which we will study in a later chapter).

These special products can be shown as the **squares of a binomial**

$(a+b)^2 \quad$ and $\quad (a-b)^2$

and as the **sum and difference of two terms**.

$(a+b)\,(a-b)$

There are two simple rules that allow us to easily expand (multiply out) these binomials. They are well worth memorizing, as they will save a lot of time in the future.

Expanding $(a+b)^2$ and $(a-b)^2$

Squaring a Binomial
To square a binomial:*

1. Square the first term.
2. Take the product of the two terms and double it.
3. Square the last term.
4. Add the three results together.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Expanding $(a+b)\,(a-b)$

Sum and Difference of Two Terms
To expand the sum and difference of two terms:†

1. Square the first term and square the second term.
2. Subtract the square of the second term from the square of the first term.

$$(a+b)(a-b)=a^2-b^2$$

* See problems 56 and 57 at the end of this section.
† See problem 58.

Sample Set A

Example:

$(x+4)^2$

$\text{Square the first term:} \quad x^2.$
$\text{The product of both terms is } 4x. \text{ Double it:} \quad 8x.$
$\text{Square the last term:} \quad 16.$
$\text{Add them together:} \quad x^2+8x+16.$

$(x+4)^2=x^2+8x+16$
Note that $(x+4)^2 \neq x^2+4^2$. The $8x$ term is missing!

Example:

$(a-8)^2$

$\text{Square the first term:} \quad a^2.$
$\text{The product of both terms is } -8a. \text{ Double it:} \quad -16a.$
$\text{Square the last term:} \quad 64.$
$\text{Add them together:} \quad a^2+(-16a)+64.$

$(a-8)^2=a^2-16a+64$
Notice that the sign of the last term in this expression is "+." This will always happen since the last term results from a number being **squared**. Any nonzero number times itself is always positive.
$(+)(+)=+ \quad \text{and} \quad (-)(-)=+$
The sign of the second term in the trinomial will always be the sign that occurs **inside** the parentheses.

Example:

$(y-1)^2$ Square the first term: y^2.

The product of both terms is $-y$. Double it: $-2y$.

Square the last term: $+1$.

Add them together: $y^2 + (-2y) + 1$.

$$(y-1)^2 = y^2 \underset{\text{minus}}{-} 2y \underset{\text{plus}}{+} 1$$

Example:

$(5x+3)^2$ Square the first term: $25x^2$.

The product of both terms is $15x$. Double it: $30x$.

Square the last term: 9.

Add them together: $25x^2 + 30x + 9$.

$$(5x+3)^2 = 25x^2 \underset{\text{plus}}{+} 30x \underset{\text{plus}}{+} 9$$

Example:

$(7b-2)^2$ Square the first term: $49b^2$.

The product of both terms is $-14b$. Double it: $-28b$.

Square the last term: 4.

Add them together: $49b^2 + (-28b) + 4$.

$$(7b-2)^2 = 49b^2 \underset{\text{minus}}{-} 28b \underset{\text{plus}}{+} 4$$

Example:

$(x+6)(x-6)$ Square the first term: x^2.

Subtract the square of the second term (36) from the square of the first term: $x^2 - 36$.

$(x+6)(x-6) = x^2 - 36$

Example:

$(4a-12)(4a+12)$

Square the first term: $16a^2$.

Subtract the square of the second term (144) from the square of the first term: $16a^2-144$.

$(4a-12)(4a+12)=16a^2-144$

Example:

$(6x+8y)(6x-8y)$

Square the first term: $36x^2$.

Subtract the square of the second term $(64y^2)$ from the square of the first term: $36x^2-64y^2$.

$(6x+8y)(6x-8y)=36x^2-64y^2$

Practice Set A

Find the following products.

Exercise:

Problem: $(x+5)^2$

Solution:

$x^2+10x+25$

Exercise:

Problem: $(x+7)^2$

Solution:

$x^2+14x+49$

Exercise:

Problem: $(y-6)^2$

Solution:

$y^2-12y+36$

Exercise:

Problem: $(3a+b)^2$

Solution:

$9a^2+6ab+b^2$

Exercise:

Problem: $(9m-n)^2$

Solution:

$81m^2-18mn+n^2$

Exercise:

Problem: $(10x-2y)^2$

Solution:

$100x^2-40xy+4y^2$

Exercise:

Problem: $(12a-7b)^2$

Solution:

$144a^2-168ab+49b^2$

Exercise:

Problem: $(5h-15k)^2$

Solution:

$25h^2-150hk+225k^2$

Exercises

For the following problems, find the products.

Exercise:

Problem: $(x+3)^2$

Solution:

$x^2 + 6x + 9$

Exercise:

Problem: $(x + 5)^2$

Exercise:

Problem: $(x + 8)^2$

Solution:

$x^2 + 16x + 64$

Exercise:

Problem: $(x + 6)^2$

Exercise:

Problem: $(y + 9)^2$

Solution:

$y^2 + 18y + 81$

Exercise:

Problem: $(y + 1)^2$

Exercise:

Problem: $(a - 4)^2$

Solution:

$a^2 - 8a + 16$

Exercise:

Problem: $(a - 6)^2$

Exercise:

Problem: $(a - 7)^2$

Solution:

$a^2 - 14a + 49$

Exercise:

Problem: $(b + 10)^2$

Exercise:

Problem: $(b + 15)^2$

Solution:

$b^2 + 30b + 225$

Exercise:

Problem: $(a - 10)^2$

Exercise:

Problem: $(x - 12)^2$

Solution:

$x^2 - 24x + 144$

Exercise:

Problem: $(x + 20)^2$

Exercise:

Problem: $(y - 20)^2$

Solution:

$y^2 - 40y + 400$

Exercise:

Problem: $(3x + 5)^2$

Exercise:

Problem: $(4x+2)^2$

Solution:

$16x^2 + 16x + 4$

Exercise:

Problem: $(6x-2)^2$

Exercise:

Problem: $(7x-2)^2$

Solution:

$49x^2 - 28x + 4$

Exercise:

Problem: $(5a-6)^2$

Exercise:

Problem: $(3a-9)^2$

Solution:

$9a^2 - 54a + 81$

Exercise:

Problem: $(3w-2z)^2$

Exercise:

Problem: $(5a-3b)^2$

Solution:

$25a^2 - 30ab + 9b^2$

Exercise:

Problem: $(6t-7s)^2$

Exercise:

Problem: $(2h - 8k)^2$

Solution:

$4h^2 - 32hk + 64k^2$

Exercise:

Problem: $\left(a + \frac{1}{2}\right)^2$

Exercise:

Problem: $\left(a + \frac{1}{3}\right)^2$

Solution:

$a^2 + \frac{2}{3}a + \frac{1}{9}$

Exercise:

Problem: $\left(x + \frac{3}{4}\right)^2$

Exercise:

Problem: $\left(x + \frac{2}{5}\right)^2$

Solution:

$x^2 + \frac{4}{5}x + \frac{4}{25}$

Exercise:

Problem: $\left(x - \frac{2}{3}\right)^2$

Exercise:

Problem: $\left(y - \frac{5}{6}\right)^2$

Solution:

$y^2 - \frac{5}{3}y + \frac{25}{36}$

Exercise:

Problem: $\left(y+\frac{2}{3}\right)^2$

Exercise:

Problem: $(x+1.3)^2$

Solution:

$x^2+2.6x+1.69$

Exercise:

Problem: $(x+5.2)^2$

Exercise:

Problem: $(a+0.5)^2$

Solution:

$a^2+a+0.25$

Exercise:

Problem: $(a+0.08)^2$

Exercise:

Problem: $(x-3.1)^2$

Solution:

$x^2-6.2x+9.61$

Exercise:

Problem: $(y-7.2)^2$

Exercise:

Problem: $(b-0.04)^2$

Solution:

$b^2-0.08b+0.0016$

Exercise:

Problem: $(f - 1.006)^2$

Exercise:

Problem: $(x + 5)(x - 5)$

Solution:

$x^2 - 25$

Exercise:

Problem: $(x + 6)(x - 6)$

Exercise:

Problem: $(x + 1)(x - 1)$

Solution:

$x^2 - 1$

Exercise:

Problem: $(t - 1)(t + 1)$

Exercise:

Problem: $(f + 9)(f - 9)$

Solution:

$f^2 - 81$

Exercise:

Problem: $(y - 7)(y + 7)$

Exercise:

Problem: $(2y + 3)(2y - 3)$

Solution:

$4y^2 - 9$

Exercise:

Problem: $(5x+6)(5x-6)$

Exercise:

Problem: $(2a-7b)(2a+7b)$

Solution:

$4a^2-49b^2$

Exercise:

Problem: $(7x+3t)(7x-3t)$

Exercise:

Problem: $(5h-2k)(5h+2k)$

Solution:

$25h^2-4k^2$

Exercise:

Problem: $\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right)$

Exercise:

Problem: $\left(a+\frac{2}{9}\right)\left(a-\frac{2}{9}\right)$

Solution:

$a^2-\frac{4}{81}$

Exercise:

Problem: $\left(x+\frac{7}{3}\right)\left(x-\frac{7}{3}\right)$

Exercise:

Problem: $\left(2b+\frac{6}{7}\right)\left(2b-\frac{6}{7}\right)$

Solution:

$4b^2 - \frac{36}{49}$

Exercise:

Problem: Expand $(a+b)^2$ to prove it is equal to $a^2 + 2ab + b^2$.

Exercise:

Problem: Expand $(a-b)^2$ to prove it is equal to $a^2 - 2ab + b^2$.

Solution:

$(a-b)(a-b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$

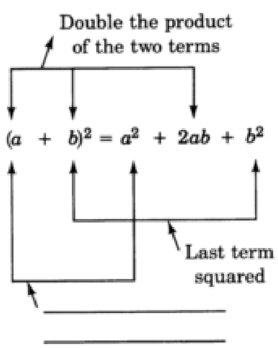
Exercise:

Problem: Expand $(a+b)(a-b)$ to prove it is equal to $a^2 - b^2$.

Exercise:

Fill in the missing label in the equation below.

Problem:



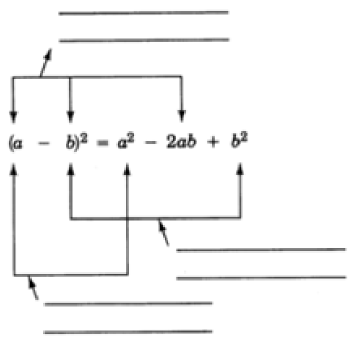
Solution:

first term squared

Exercise:

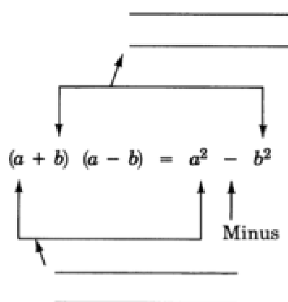
Label the parts of the equation below.

Problem:



Exercise:

Problem: Label the parts of the equation below.



Solution:

(a) Square the first term.
(b) Square the second term and subtract it from the first term.

Exercises for Review

Exercise:

Problem: ([link]) Simplify $\left(x^3y^0z^4\right)^5$.

Exercise:

Problem: ([link]) Find the value of $10^{-1} \cdot 2^{-3}$.

Solution:

$\frac{1}{80}$

Exercise:

Problem: (**[link]**) Find the product. $(x+6)(x-7)$.

Exercise:

Problem: (**[link]**) Find the product. $(5m-3)(2m+3)$.

Solution:

$10m^2+9m-9$

Exercise:

Problem: (**[link]**) Find the product. $(a+4)(a^2-2a+3)$.

Division of Polynomials
<para>This module is from <link document="col10614">Elementary Algebra</link> by Denny Burzynski and Wade Ellis, Jr.</para> <para>A detailed study of arithmetic operations with rational expressions is presented in this chapter, beginning with the definition of a rational expression and then proceeding immediately to a discussion of the domain. The process of reducing a rational expression and illustrations of multiplying, dividing, adding, and subtracting rational expressions are also included. Since the operations of addition and subtraction can cause the most difficulty, they are given particular attention. We have tried to make the written explanation of the examples clearer by using a "freeze frame" approach, which walks the student through the operation step by step.</para> <para>The five-step method of solving applied problems is included in this chapter to show the problem-solving approach to number problems, work problems, and geometry problems. The chapter also illustrates simplification of complex rational expressions, using the combine-divide method and the LCD-multiply-divide method.</para> <para>Objectives of this module: be able to divide a polynomial by a monomial, understand the process and be able to divide a polynomial by a polynomial.</para>

Overview

- Dividing a Polynomial by a Monomial
- The Process of Division
- Review of Subtraction of Polynomials
- Dividing a Polynomial by a Polynomial

Dividing A Polynomial By A Monomial

The following examples illustrate how to divide a polynomial by a monomial. The division process is quite simple and is based on addition of rational expressions.

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

Turning this equation around we get

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Now we simply divide c into a, and c into b. This should suggest a rule.

Dividing a Polynomial By a Monomial
To divide a polynomial by a monomial, divide every term of the polynomial by the monomial.

Sample Set A

Example:
$\frac{3x^2+x-11}{x}$. Divide every term of $3x^2 + x - 11$ by x.

$$\frac{3x^2}{x} + \frac{x}{x} - \frac{11}{x} = 3x + 1 - \frac{11}{x}$$

Example:
$\frac{8a^3+4a^2-16a+9}{2a^2}$. Divide every term of $8a^3 + 4a^2 - 16a + 9$ by $2a^2$.

$$\frac{8a^3}{2a^2} + \frac{4a^2}{2a^2} - \frac{16a}{2a^2} + \frac{9}{2a^2} = 4a + 2 - \frac{8}{a} + \frac{9}{2a^2}$$

Example:
$\frac{4b^6-9b^4-2b+5}{-4b^2}$. Divide every term of $4b^6 - 9b^4 - 2b + 5$ by $-4b^2$.

$$\frac{4b^6}{-4b^2} - \frac{9b^4}{-4b^2} - \frac{2b}{-4b^2} + \frac{5}{-4b^2} = -b^4 + \frac{9}{4}b^2 + \frac{1}{2b} - \frac{5}{4b^2}$$

Practice Set A

Perform the following divisions.

Exercise:

Problem: $\frac{2x^2+x-1}{x}$

Solution:

$2x+1-\frac{1}{x}$

Exercise:

Problem: $\frac{3x^3+4x^2+10x-4}{x^2}$

Solution:

$3x+4+\frac{10}{x}-\frac{4}{x^2}$

Exercise:

Problem: $\frac{a^2b+3ab^2+2b}{ab}$

Solution:

$a+3b+\frac{2}{a}$

Exercise:

Problem: $\frac{14x^2y^2-7xy}{7xy}$

Solution:

$2xy-1$

Exercise:

Problem: $\frac{10m^3n^2+15m^2n^3-20mn}{-5m}$

Solution:

$$-2m^2n^2 - 3mn^3 + 4n$$

The Process Of Division

In Section [link] we studied the method of reducing rational expressions. For example, we observed how to reduce an expression such as

$$\frac{x^2-2x-8}{x^2-3x-4}$$

Our method was to factor both the numerator and denominator, then divide out common factors.

$$\frac{(x-4)(x+2)}{(x-4)(x+1)}$$

$$\frac{\cancel{(x-4)}\,(x+2)}{\cancel{(x-4)}\,(x+1)}$$

$$\frac{x+2}{x+1}$$

When the numerator and denominator have no factors in common, the division may still occur, but the process is a little more involved than merely factoring. The method of dividing one polynomial by another is much the same as that of dividing one number by another. First, we'll review the steps in dividing numbers.

1. $\frac{35}{8}$. We are to divide 35 by 8.
2. $8\overline{)35.}$

   We try 4, since 32 divided by 8 is 4.

3. $\begin{array}{r} 4 \\ 8\overline{)35.} \end{array}$

   Multiply 4 and 8.

4. $\begin{array}{r} 4 \\ 8\overline{)35.} \\ 32 \end{array}$

   Subtract 32 from 35.

5. $\begin{array}{r} 4 \\ 8\overline{)35.} \\ \underline{32} \\ 3 \end{array}$

   Since the remainder 3 is less than the divisor 8, we are done with the 32 division.

6. $4\frac{3}{8}$. The quotient is expressed as a mixed number.

The process was to divide, multiply, and subtract.

Review Of Subtraction Of Polynomials

A very important step in the process of dividing one polynomial by another is subtraction of polynomials. Let's review the process of subtraction by observing a few examples.

1. Subtract $x-2$ from $x-5$; that is, find $(x-5)-(x-2)$.

Since $\mathrm{x}-2$ is preceded by a minus sign, remove the parentheses, change the sign of each term, then add.

$$\begin{array}{r} \mathrm{x}-5 \\ \underline{-(x-2)} \end{array} = \begin{array}{r} x-5 \\ \underline{-x+2} \\ -3 \end{array}$$

The result is -3.

2. Subtract x^3+3x^2 from x^3+4x^2+x-1.

Since $x^3 + 3x^2$ is preceded by a minus sign, remove the parentheses, change the sign of each term, then add.

$$\frac{\begin{array}{l} x^3+4x^2+x-1 \\ -(x^3+3x^2) \end{array}}{} = \frac{\begin{array}{l} x^3+4x^2+x-1 \\ -x^3-3x^2 \end{array}}{x^2+x-1}$$

The result is $x^2 + x - 1$.

3. Subtract $x^2 + 3x$ from $x^2 + 1$.

We can write $x^2 + 1$ as $x^2 + 0x + 1$.

$$\frac{\begin{array}{l} x^2+1 \\ -(x^2+3x) \end{array}}{} = \frac{\begin{array}{l} x^2+0x+1 \\ -(x^2+3x) \end{array}}{} = \frac{\begin{array}{l} x^2+0x+1 \\ -x^2-3x \end{array}}{-3x+1}$$

Dividing A Polynomial By A Polynomial

Now we'll observe some examples of dividing one polynomial by another. The process is the same as the process used with whole numbers: divide, multiply, subtract, divide, multiply, subtract,....

The division, multiplication, and subtraction take place one term at a time. The process is concluded when the polynomial remainder is of lesser degree than the polynomial divisor.

Sample Set B

Perform the division.

Example:

$\frac{x-5}{x-2}$. We are to divide $x - 5$ by $x - 2$.

$x-2\overline{)\,x-5}$	**Divide x into x.**
$\begin{array}{r} 1 \\ \boxed{x}-2\overline{)\,x-5} \end{array}$	**Multiply 1 and $x-2$.**
$\begin{array}{r} 1 \\ x-2\overline{)\,x-5} \\ x-2 \end{array}$	**Multiply 1 and $x-2$.** **Subtract $x-2$ from $x-5$.**
$\begin{array}{r} 1 \\ x-2\overline{)\,x-5} \\ \underline{-x+2} \\ -3 \end{array}$	**We write the quotient as**

$1-\frac{3}{x-2}$

Thus,

$\frac{x-5}{x-2}=1-\frac{3}{x-2}$

Example:

$\frac{x^3+4x^2+x-1}{x+3}$. We are to divide x^3+4x^2+x-1 by $x+3$.

$$x+3\overline{\smash{)}x^3+4x^2+x-1}$$

Divide x into x^3.

$$\begin{array}{r} x^2 \\ \boxed{x}+3\overline{\smash{)}x^3+4x^2+x-1} \end{array}$$

Multiply x^2 and $x+3$.

$$\begin{array}{r} x^2 \\ x+3\overline{\smash{)}x^3+4x^2+x-1} \\ x^3+3x^2 \end{array}$$

Multiply x^2 and $x+3$.
Subtract x^3+3x^2 from x^3+4x^2+x-1.

$$\begin{array}{r} x^2 \\ x+3\overline{\smash{)}x^3+4x^2+x-1} \\ \underline{{}^{-}x^3 \not+ 3x^2} \\ x^2+x-1 \end{array}$$

Now divide x into x^2.

$$\begin{array}{r} x^2+x \\ \boxed{x}+3\overline{\smash{)}x^3+4x^2+x-1} \\ \underline{x^3+3x^2} \\ \longrightarrow x^2+x-1 \end{array}$$

Multiply x and $x+3$.

$$\begin{array}{r} x^2+x \\ x+3\overline{\smash{)}x^3+4x^2+\ \ x-1} \\ \underline{x^3+3x^3} \\ x^2+\ \ x-1 \\ x^2+3x \end{array}$$

Multiply x and $x+3$.
Subtract x^2+3x from x^2+x-1.

$$\begin{array}{r} x^2+x \\ x+3\overline{\smash{)}x^3+4x^2+\ \ x-1} \\ \underline{x^3+3x^3} \\ x^2+\ \ x-1 \\ \underline{{}^{-}x^2 \not+ 3x} \\ -2x-1 \end{array}$$

Divide x into $-2x$.

$$\begin{array}{r} x^2+x-2 \\ \boxed{x}+3\overline{\smash{)}x^3+4x^2+\ \ x-1} \\ \underline{x^3+3x^2} \\ x^2+\ \ x-1 \\ \underline{x^2+3x} \\ \longrightarrow -2x-1 \end{array}$$

$$\begin{array}{r} x^2+x-2 \\ x+3\overline{\smash{)}x^3+4x^2+\ \ x-1} \\ \underline{x^3+3x^2} \\ x^2+\ \ x-1 \\ \underline{x^2+3x} \\ -2x-1 \\ -2x-6 \end{array}$$

Multiply -2 and $x+3$.
Subtract $-2x-6$ from $-2x-1$.

$$\begin{array}{r} x^2+x-2 \\ x+3\overline{\smash{)}x^3+4x^2+\ \ x-1} \\ \underline{x^3+3x^2} \\ x^2+\ \ x-1 \\ \underline{x^2+3x} \\ -2x-1 \\ {}^{+}\underline{-2x\pm 6} \\ 5 \end{array}$$

Since the polynomial 5 is of lesser degree than $x+3$ $(0<1)$, we are finished. We write the quotient as

$x^2 + x - 2 + \frac{5}{x+3}$

Thus,

$\frac{x^3+4x^2+x-1}{x+3} = x^2 + x - 2 + \frac{5}{x+3}$

Practice Set B

Perform the following divisions.

Exercise:

Problem: $\frac{x+6}{x-1}$

Solution:

$1 + \frac{7}{x-1}$

Exercise:

Problem: $\frac{x^2+2x+5}{x+3}$

Solution:

$x - 1 + \frac{8}{x+3}$

Exercise:

Problem: $\frac{x^3+x^2-x-2}{x+8}$

Solution:

$x^2 - 7x + 55 - \frac{442}{x+8}$

Exercise:

Problem: $\frac{x^3+x^2-3x+1}{x^2+4x-5}$

Solution:

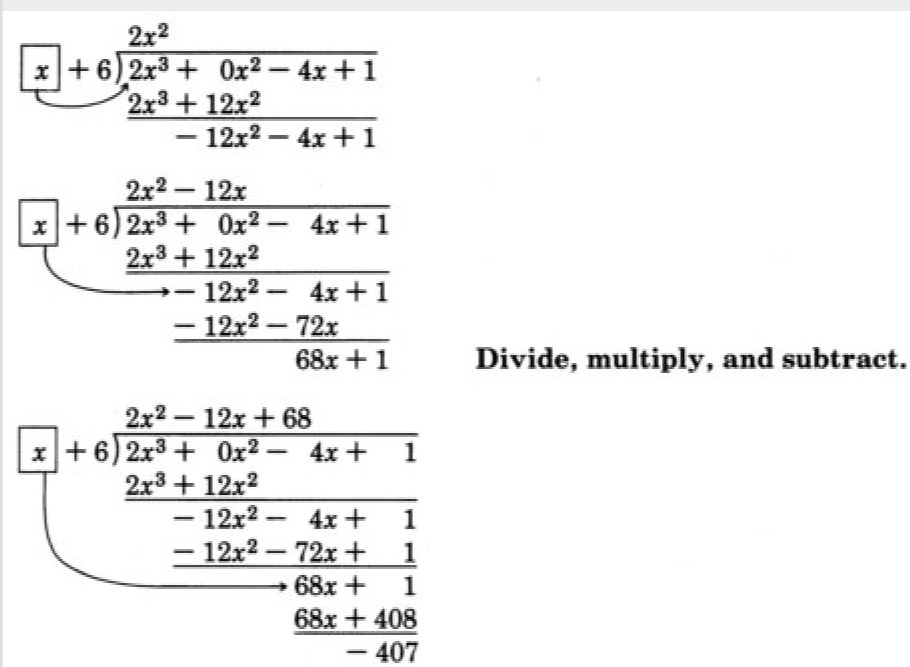
$$x-3+\frac{14x-14}{x^2+4x-5}=x-3+\frac{14}{x+5}$$

Sample Set C

Example:

Divide $2x^3-4x+1$ by $x+6$.

$\frac{2x^3-4x+1}{x+6}$	Notice that the x^2 term in the numerator is missing. We can avoid any confusion by writing
$\frac{2x^3+0x^2-4x+1}{x+6}$	Divide, multiply, and subtract.



$\frac{2x^3-4x+1}{x+6} = 2x^3 - 12x + 68 - \frac{407}{x+6}$

Practice Set C

Perform the following divisions.

Exercise:

Problem: $\frac{x^2-3}{x+2}$

Solution:

$x - 2 + \frac{1}{x+2}$

Exercise:

Problem: $\frac{4x^2-1}{x-3}$

Solution:

$4x + 12 + \frac{35}{x-3}$

Exercise:

Problem: $\frac{x^3+2x+2}{x-2}$

Solution:

$x^2 + 2x + 6 + \frac{14}{x-2}$

Exercise:

Problem: $\frac{6x^3+5x^2-1}{2x+3}$

Solution:

$3x^2 - 2x + 3 - \frac{10}{2x+3}$

Exercises

For the following problems, perform the divisions.

Exercise:

Problem: $\frac{6a+12}{2}$

Solution:

$3a + 6$

Exercise:

Problem: $\frac{12b-6}{3}$

Exercise:

Problem: $\frac{8y-4}{-4}$

Solution:

$-2y + 1$

Exercise:

Problem: $\frac{21a-9}{-3}$

Exercise:

Problem: $\frac{3x^2-6x}{-3}$

Solution:

$-x\,(x-2)$

Exercise:

Problem: $\frac{4y^2-2y}{2y}$

Exercise:

Problem: $\frac{9a^2+3a}{3a}$

Solution:

$3a+1$

Exercise:

Problem: $\frac{20x^2+10x}{5x}$

Exercise:

Problem: $\frac{6x^3+2x^2+8x}{2x}$

Solution:

$3x^2+x+4$

Exercise:

Problem: $\frac{26y^3+13y^2+39y}{13y}$

Exercise:

Problem: $\frac{a^2b^2+4a^2b+6ab^2-10ab}{ab}$

Solution:

$ab + 4a + 6b - 10$

Exercise:

Problem: $\frac{7x^3y+8x^2y^3+3xy^4-4xy}{xy}$

Exercise:

Problem: $\frac{5x^3y^3-15x^2y^2+20xy}{-5xy}$

Solution:

$-x^2y^2 + 3xy - 4$

Exercise:

Problem: $\frac{4a^2b^3-8ab^4+12ab^2}{-2ab^2}$

Exercise:

Problem: $\frac{6a^2y^2+12a^2y+18a^2}{24a^2}$

Solution:

$\frac{1}{4}y^2 + \frac{1}{2}y + \frac{3}{4}$

Exercise:

Problem: $\frac{3c^3y^3+99c^3y^4-12c^3y^5}{3c^3y^3}$

Exercise:

Problem: $\frac{16ax^2-20ax^3+24ax^4}{6a^4}$

Solution:

$\frac{8x^2-10x^3+12x^4}{3a^3}$ or $\frac{12x^4-10x^3+8x^2}{3a^3}$

Exercise:

Problem: $\frac{21ay^3-18ay^2-15ay}{6ay^2}$

Exercise:

Problem: $\frac{-14b^2c^2+21b^3c^3-28c^3}{-7a^2c^3}$

Solution:

$\frac{2b^2-3b^3c+4c}{a^2c}$

Exercise:

Problem: $\frac{-30a^2b^4-35a^2b^3-25a^2}{-5b^3}$

Exercise:

Problem: $\frac{x+6}{x-2}$

Solution:

$1+\frac{8}{x-2}$

Exercise:

Problem: $\frac{y+7}{y+1}$

Exercise:

Problem: $\frac{x^2-x+4}{x+2}$

Solution:

$$x - 3 + \frac{10}{x+2}$$

Exercise:

Problem: $\frac{x^2+2x-1}{x+1}$

Exercise:

Problem: $\frac{x^2-x+3}{x+1}$

Solution:

$$x - 2 + \frac{5}{x+1}$$

Exercise:

Problem: $\frac{x^2+5x+5}{x+5}$

Exercise:

Problem: $\frac{x^2-2}{x+1}$

Solution:

$$x - 1 - \frac{1}{x+1}$$

Exercise:

Problem: $\frac{a^2-6}{a+2}$

Exercise:

Problem: $\frac{y^2+4}{y+2}$

Solution:

$y-2+\frac{8}{y+2}$

Exercise:

Problem: $\frac{x^2+36}{x+6}$

Exercise:

Problem: $\frac{x^3-1}{x+1}$

Solution:

$x^2-x+1-\frac{2}{x+1}$

Exercise:

Problem: $\frac{a^3-8}{a+2}$

Exercise:

Problem: $\frac{x^3-1}{x-1}$

Solution:

x^2+x+1

Exercise:

Problem: $\frac{a^3-8}{a-2}$

Exercise:

Problem: $\frac{x^3+3x^2+x-2}{x-2}$

Solution:

$x^2 + 5x + 11 + \frac{20}{x-2}$

Exercise:

Problem: $\frac{a^3+2a^2-a+1}{a-3}$

Exercise:

Problem: $\frac{a^3+a+6}{a-1}$

Solution:

$a^2 + a + 2 + \frac{8}{a-1}$

Exercise:

Problem: $\frac{x^3+2x+1}{x-3}$

Exercise:

Problem: $\frac{y^3+3y^2+4}{y+2}$

Solution:

$y^2 + y - 2 + \frac{8}{y+2}$

Exercise:

Problem: $\frac{y^3+5y^2-3}{y-1}$

Exercise:

Problem: $\frac{x^3+3x^2}{x+3}$

Solution:

x^2

Exercise:

Problem: $\frac{a^2+2a}{a+2}$

Exercise:

Problem: $\frac{x^2-x-6}{x^2-2x-3}$

Solution:

$1 + \frac{1}{x+1}$

Exercise:

Problem: $\frac{a^2+5a+4}{a^2-a-2}$

Exercise:

Problem: $\frac{2y^2+5y+3}{y^2-3y-4}$

Solution:

$2 + \frac{11}{y-4}$

Exercise:

Problem: $\frac{3a^2+4a-4}{a^2+3a+3}$

Exercise:

Problem: $\frac{2x^2-x+4}{2x-1}$

Solution:

$x+\frac{4}{2x-1}$

Exercise:

Problem: $\frac{3a^2+4a+2}{3a+4}$

Exercise:

Problem: $\frac{6x^2+8x-1}{3x+4}$

Solution:

$2x-\frac{1}{3x+4}$

Exercise:

Problem: $\frac{20y^2+15y-4}{4y+3}$

Exercise:

Problem: $\frac{4x^3+4x^2-3x-2}{2x-1}$

Solution:

$2x^2+3x-\frac{2}{2x-1}$

Exercise:

Problem: $\frac{9a^3-18a^2+8a-1}{3a-2}$

Exercise:

Problem: $\frac{4x^4-4x^3+2x^2-2x-1}{x-1}$

Solution:

$4x^3 + 2x - \frac{1}{x-1}$

Exercise:

Problem: $\frac{3y^4+9y^3-2y^2-6y+4}{y+3}$

Exercise:

Problem: $\frac{3y^2+3y+5}{y^2+y+1}$

Solution:

$3 + \frac{2}{y^2+y+1}$

Exercise:

Problem: $\frac{2a^2+4a+1}{a^2+2a+3}$

Exercise:

Problem: $\frac{8z^6-4z^5-8z^4+8z^3+3z^2-14z}{2z-3}$

Solution:

$4z^5 + 4z^4 + 2z^3 + 7z^2 + 12z + 11 + \frac{33}{2z-3}$

Exercise:

Problem: $\frac{9a^7+15a^6+4a^5-3a^4-a^3+12a^2+a-5}{3a+1}$

Exercise:

Problem: $(2x^5 + 5x^4 - 1) \div (2x + 5)$

Solution:

$x^4 - \frac{1}{2x+5}$

Exercise:

Problem:$(6a^4 - 2a^3 - 3a^2 + a + 4) \div (3a - 1)$

Exercises For Review

Exercise:

Problem:([link]) Find the product. $\frac{x^2+2x-8}{x^2-9} \cdot \frac{2x+6}{4x-8}$.

Solution:

$\frac{x+4}{2(x-3)}$

Exercise:

Problem:([link]) Find the sum. $\frac{x-7}{x+5} + \frac{x+4}{x-2}$.

Exercise:

Problem:([link]) Solve the equation $\frac{1}{x+3} + \frac{1}{x-3} = \frac{1}{x^2-9}$.

Solution:

$x = \frac{1}{2}$

Exercise:

Problem:

([link]) When the same number is subtracted from both the numerator and denominator of $\frac{3}{10}$, the result is $\frac{1}{8}$. What is the number that is subtracted?

Exercise:

Problem:([link]) Simplify $\frac{\frac{1}{x+5}}{\frac{4}{x^2-25}}$.

Solution:

$\frac{x-5}{4}$